

Non-stationary Multicomponent Signals Analysis using the Synchrosqueezing Transform

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Plan

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- 2 Time-frequency analysis in a nutshell
- 3 The synchrosqueezing method
 - The reassignment method
 - Synchrosqueezing in a nutshell
 - Components extraction
- 4 New research directions
 - Practical applications
 - Theoretical works

A short bio...

MCF61 IUT d'Evry, dept TJC, (qualif. CNU 26, 27, 61).

Postes occupés

- 2016-2018 : postdoc à l'IRCAM (H2020 ABC-DJ)
- 2015-2016 : postdoc à l'IREENA (Saint-Nazaire) + LJK (Grenoble) projet ANR ASTRES (1 an + 6 mois mi-temps)
- 2014-2015 : postdoc au LaBRI, projet ANR DIADEMS
- 2009-2013 : doctorant+ATER au LaBRI, université de Bordeaux 1
- avant 2009 : ingénieur R&D : SC2X S.A.S (stage Bordeaux), Sisenior SPRL (développeur freelance, Bruxelles, BE),

Formation

- Doctorat informatique : "Approche informée pour l'analyse du son et de la musique" (direc. S. Marchand). dec. 2013. (EDMI, univ. Bordeaux 1)
- Master 2 mathématiques appliquées : "Traitement harmonique et contrôle du signal" (univ. Bordeaux 1)
- Master 2 informatique : "Image et son, multimédia" (univ. Bordeaux 1)

Expériences et compétences

Thèmes de recherche

- Traitement du signal : analyse temps-fréquence, analyse spectrale, codage de sources, problèmes inverses, séparation de sources
- Extraction d'informations musicales (MIR) : estimation F_0 multiple, reconnaissance du timbre instrumental, apprentissage machine
- Traitement de la parole : caractérisation et modélisation des attitudes socio-culturelles (analyse statistique et apprentissage machine)

Projets de recherche

- H2020, ABC-DJ (IRCAM *et al.* , 2016-2018)
- ANR ASTRES (ENS LYON, LJK, IREENA, 2013-2017)
- ANR DIADEMS (IRIT, LaBRI, LIMSI, LAM, CREM, 2012-2015)
- PEPS/IDEX, univ. Bordeaux (2013-2014)
- ANR DReAM (LaBRI, Lab-STICC, TSI, GIPSA-lab, 2009-2014)

Publications

Plus de 30 articles publiés dans des revues (5+) et conférences internationales (25+).

Tâches collectives et vie associative

Activités de relecture

- Journaux internationaux : IEEE TSP, SPL, JASA. Math. Geosciences
- Conférences internationales : ICASSP, ISMIR, EUSIPCO, DAFx, ...

Tâches collectives

- Membre IEEE, de l'AFCP (Association Francophone de la Communication Parlée),
- Membre du LaBRUIT (jusqu'en 2013),
- Développeur et webmaster du site de l'EDMI (école doctorale maths-info) de Bordeaux,
- Développeur et webmaster du site du projet ANR DREAM.

Divers

- Musicien de jazz (piano), formation conservatoire (CRR Bordeaux)
- Associé, YES WE CARE SPRL (www.myspecialist.be), BE 0845.536.330 -
Siège social : Avenue Grandchamp, 93 - 1150 Bruxelles, Belgique.

Purpose of this presentation

Goals

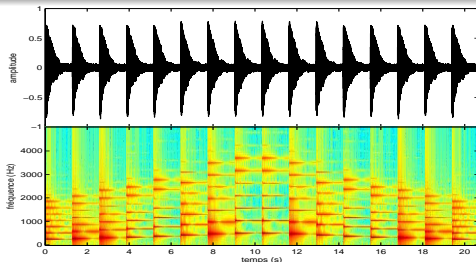
- Meaningful **information extraction** from observation (signal or image) .
- Computing efficient representations designed for **classification/detection/prediction** problems.
- **Separation/clustering** of elementary signal components (atoms).

Investigated approaches

- **Time-frequency/Time-scale** reassignment-based transformations (i.e. synchrosqueezing)
- ~~Data-driven empirical signal decomposition (e.g. EMD, SSA)~~ ⇒ in a future presentation.

⇒ rationales and open research directions

Why time-frequency analysis (TFA)?



Waveform and spectrogram of a piano playing the C major scale.

Motivations : non-stationary multicomponent signals analysis/modeling

TFA can :

- help to disentangle/segregate elementary components (*i.e.* sinusoids, noise, transients, etc.)
- lead to sparse representation (useful for data compression)
- lead to efficient and physically meaningful representations to be combined with a machine learning approach
- provide intuitive and readable representations similar to a music score (related to "instantaneous frequency" [Ville, 48])

Principle and limitations

Linear transform of a signal x using a time-frequency decomposition

$$\Phi_x(\gamma) = \int_{-\infty}^{+\infty} x(t)\phi_\gamma^*(t) dt \quad (1)$$

with $x, \phi_\gamma \in L^2(\mathbb{R})$ et $\|\phi_\gamma\|^2 = \int_{\mathbb{R}} |\phi_\gamma(t)|^2 dt = 1$

Heisenberg-Gabor uncertainty principle (1927)

$$\sigma_t \sigma_\omega \geq \frac{1}{2} \quad (2)$$

with :

$$\sigma_t^2 = \int_{-\infty}^{+\infty} (t - \bar{t})^2 |\phi_\gamma(t)|^2 dt \text{ et } \bar{t} = \int_{\mathbb{R}} t |\phi_\gamma(t)|^2 dt$$

$$\sigma_\omega^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (\omega - \bar{\omega})^2 |\hat{\phi}_\gamma(\omega)|^2 d\omega \text{ et } \bar{\omega} = \frac{1}{2\pi} \int_{\mathbb{R}} \omega |\hat{\phi}_\gamma(\omega)|^2 d\omega$$

with $\hat{\phi}(\omega) = \int_{\mathbb{R}} \phi_\gamma(t) e^{-j\omega t} dt$, the Fourier transform of $\phi_\gamma(t)$.

Definitions 1 : Short Time Fourier Transform (STFT)

x being the analyzed signal and h differentiable analysis window h .

Analysis

$$F_x^h(t, \omega) = \int_{\mathbb{R}} x(u) h(t-u)^* e^{-j\omega u} du, \quad (3)$$

- A time-frequency representation is provided by $|F_x^h(t, \omega)|^2$ called spectrogram.
- When $h(t) = \frac{1}{2\pi\sqrt{T}} e^{-\frac{t^2}{2T^2}}$, $F_x^h(t, \omega)$ is called the gabor transform.

Reconstruction formulas

$$\hat{x}(t) = \iint_{\mathbb{R}^2} F_x^h(\tau, \omega) e^{j\omega\tau} d\tau \frac{d\omega}{2\pi} \quad (4)$$

and its simplified reconstruction formula (when $h(t_0) \neq 0, \forall t_0 \geq 0$) :

$$x(t-t_0) = \frac{1}{h(t_0)} \int_{-\infty}^{+\infty} F_x^h(t, \omega) e^{j\omega(t-t_0)} \frac{d\omega}{2\pi}. \quad (5)$$

Definitions 2 : Continuous Wavelet Transform

Continuous Wavelet Transform

For an admissible (orthogonal) mother wavelet function Ψ (i.e.

$$C_\Psi = \int_{\mathbb{R}} \frac{|\hat{\Psi}(\omega)|^2}{|\omega|} d\omega < +\infty).$$

$$W_x^\Psi(t, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{+\infty} x(\tau) \Psi\left(\frac{\tau - t}{s}\right)^* d\tau \quad (6)$$

$s = \frac{\omega_0}{\omega}$ (s being a scale and ω_0 the center frequency of Ψ). The scalogram can be computed by $|W_x(t, s)|^2$ or $|CW_x(t, \omega)|^2$.

The Morlet wavelet is given by $\Psi(t) = \frac{\pi^{-1/4}}{\sqrt{T}} e^{-\frac{t^2}{2T^2}} e^{j\omega_0 t}$,

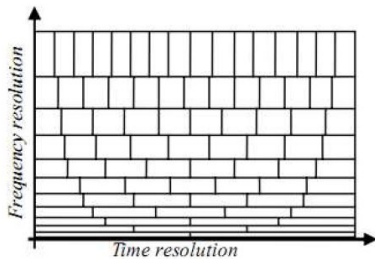
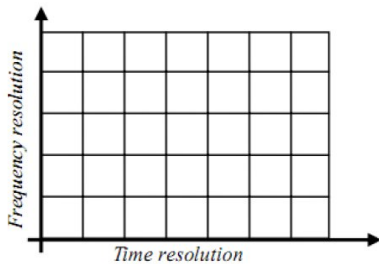
Reconstruction formula

The synthesis (Morlet) formula is given by :

$$x(t) = \frac{1}{C_\Psi} \int_{\mathbb{R}} W_x^\Psi(t, s) |s|^{-3/2} ds, \text{ with } C_\Psi = \int_{\mathbb{R}} F_\Psi(\omega)^* \frac{d\omega}{\omega} \quad (7)$$

where $F_\Psi(\omega)$ denotes the Fourier transform of $\Psi(t)$.

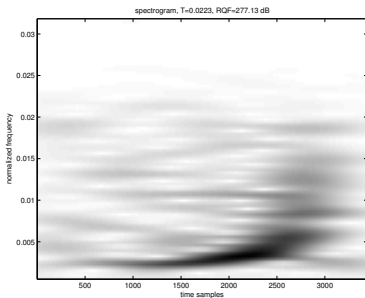
STFT vs CWT



Example : Analysis of the Livingston GW140915 signal

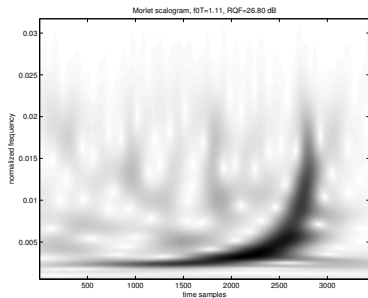


STFT



$$|F_x^h(t, \omega)|^2$$

CWT



$$|W_x(t, \omega)|^2$$

D. Fourer, J. Harmouche, J. Schmitt, T. Oberlin, S. Meignen, F. Auger and P. Flandrin. The ASTRES Toolbox for Mode Extraction of Non-Stationary Multicomponent Signals. Proc. EUSIPCO 2017, Aug. 2017. Kos Island, Greece.

The reassignment method [Kodera *et al.* 1978] [Auger & Flandrin 1995]

TF reassignment improves the energy concentration (readability) of any bilinear distribution by reassigning its energy to new locations closer to real signal support.

Considering a time-frequency representation (TFR) of a signal x expressed in terms of the Wigner-Ville distribution as :

$$\text{TFR}_x(t, \omega) = \iint_{\mathbb{R}^2} \text{WV}_x(\tau, \Omega) \Phi(t - \tau, \omega - \Omega) d\tau d\Omega$$

Method description

- Computation of the reassignment operators :

$$\hat{t}(\mathbf{t}, \omega) = \frac{\int_{\mathbb{R}^2} \tau \text{WV}_x(\tau, \Omega) \Phi(\mathbf{t} - \tau, \omega - \Omega) d\tau d\Omega}{\int_{\mathbb{R}^2} \text{WV}_x(\tau, \Omega) \Phi(\mathbf{t} - \tau, \omega - \Omega) d\tau d\Omega} \quad (8)$$

$$\hat{\omega}(\mathbf{t}, \omega) = \frac{\int_{\mathbb{R}^2} \Omega \text{WV}_x(\tau, \Omega) \Phi(\mathbf{t} - \tau, \omega - \Omega) d\tau d\Omega}{\int_{\mathbb{R}^2} \text{WV}_x(\tau, \Omega) \Phi(\mathbf{t} - \tau, \omega - \Omega) d\tau d\Omega} \quad (9)$$

- Computation of the reassigned time-frequency representation :

$$\text{RTFR}_x(\mathbf{t}, \omega) = \int_{\mathbb{R}^2} \text{TFR}_x(\tau, \Omega) \delta(\mathbf{t} - \hat{\mathbf{t}}(\tau, \Omega)) \delta(\omega - \hat{\omega}(\tau, \Omega)) d\tau d\Omega \quad (10)$$

Example : the reassigned spectrogram

$F_x^h(\mathbf{t}, \omega) = \int_{\mathbb{R}} x(\tau) h(\tau - \mathbf{t})^* e^{-j\omega\tau} d\tau$ being the STFT of a signal x using analysis window h .

$$\hat{\mathbf{t}}(\mathbf{t}, \omega) = -\frac{\partial \phi_x^h}{\partial \omega}(\mathbf{t}, \omega) = \mathbf{t} - \operatorname{Re} \left(\frac{F_x^h Th(\mathbf{t}, \omega)}{F_x^h(\mathbf{t}, \omega)} \right), \quad \text{with } Th(\mathbf{t}) = \mathbf{t} h(\mathbf{t}) \quad (11)$$

$$\hat{\omega}(\mathbf{t}, \omega) = \omega + \frac{\partial \phi_x^h}{\partial \mathbf{t}}(\mathbf{t}, \omega) = \omega + \operatorname{Im} \left(\frac{F_x^h Dh(\mathbf{t}, \omega)}{F_x^h(\mathbf{t}, \omega)} \right), \quad \text{with } Dh(\mathbf{t}) = \frac{dh}{d\mathbf{t}}(\mathbf{t}) \quad (12)$$

$$R_x(\mathbf{t}, \omega) = \iint_{\mathbb{R}^2} \left| F_x^h(\tau, \Omega) \right|^2 \delta(\mathbf{t} - \hat{\mathbf{t}}(\tau, \Omega)) \delta(\omega - \hat{\omega}(\tau, \Omega)) d\tau \frac{d\Omega}{2\pi} \quad (13)$$

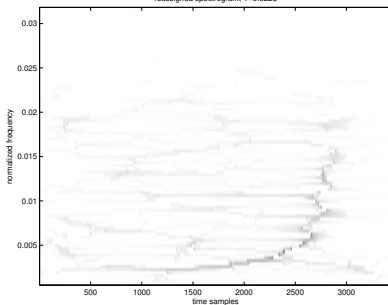
(a) $|F_x^h(t, \omega)|^2$

(b) $|R_x^h(t, \omega)|^2$

Example : TF Reassignment of the Livingston GW140915 signal

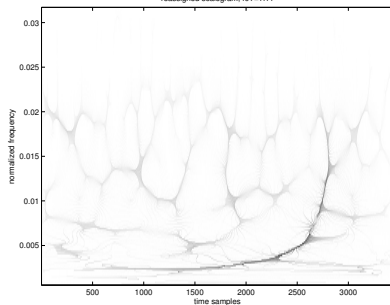
STFT

reassigned spectrogram, $T=0.0223$



CWT

reassigned scalogram, $\beta T=1.11$



Synchrosqueezing

Can be viewed as a particular reassignment method which allows to compute sharpen and reversible TFRs [Daubechies 1996, 2011] [Thakur 2011].
 Computation of the synchrosqueezed STFT and of its signal reconstruction formula :

$$\mathbf{S}_x(\mathbf{t}, \omega) = \frac{1}{h(\mathbf{0})} \int_{\mathbb{R}} F_x^h(\mathbf{t}, \Omega) \delta(\omega - \hat{\omega}(\mathbf{t}, \Omega)) \frac{d\Omega}{2\pi} \quad (14)$$

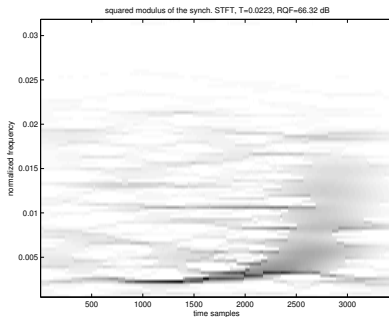
$$\hat{x}(\mathbf{t}) = \int_{\text{supp}_{\Omega}(x)} \mathbf{S}_x(\mathbf{t}, \Omega) d\Omega \quad (15)$$

(c) $|F_x^h(t, \omega)|^2$

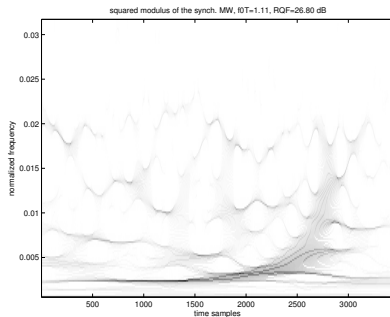
(d) $|S_x(t, \omega)|^2$

Example : Synchrosqueezing of the Livingston GW140915 signal

STFT



CWT



Second-order synchrosqueezing [Oberlin et al. 2014] [Behera et al. 2016]

Goal : to improve TFRs of strongly modulated chirps signal.

New considered signal model

$$x(t) = a(t) e^{j\Phi(t)}, \text{ with } \Phi(t) = \varphi_x + \omega_x t + \alpha_x \frac{t^2}{2}, \quad (16)$$

Enhanced IF estimation

Local modulation estimation :

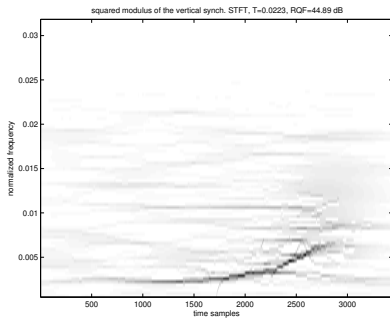
$$\hat{\alpha}_x(t, \omega) = \frac{\frac{\partial \hat{\omega}(t, \omega)}{\partial t}}{\frac{\partial \hat{t}(t, \omega)}{\partial t}} = \text{Im} \left(\frac{F_x^{\mathcal{D}^2} h(t, \omega) F_x^h(t, \omega) - F_x^{\mathcal{D}h}(t, \omega) F_x^{\mathcal{D}^2 h}(t, \omega)}{F_x^{\mathcal{D}h}(t, \omega) F_x^{\mathcal{T}h}(t, \omega) - F_x^{\mathcal{T}^{\mathcal{D}h}(t, \omega) F_x^h(t, \omega)} \right) \quad (17)$$

Enhanced IF estimator :

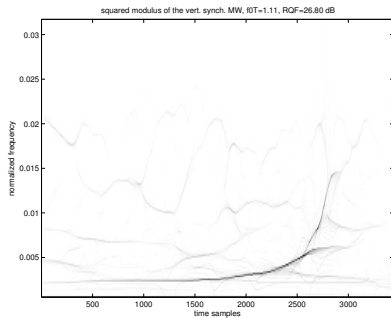
$$\hat{\omega}^{(2)}(t, \omega) = \begin{cases} \hat{\omega}(t, \omega) + \hat{\alpha}_x(t, \omega)(t - \hat{t}(t, \omega)) & \text{if } |\hat{\alpha}_x(t, \omega)| < \infty \\ \hat{\omega}(t, \omega) & \text{otherwise,} \end{cases} \quad (18)$$

Example : Vertical synchrosqueezing of the Livingston GW140915 signal

STFT



CWT



Mode extraction [Brevdo, 2011]

Goal : Automatic extraction of signal components.

Considering the synchrosqueezing transform of a signal $S_x(t, \omega)$, this technique finds the best frequency curve $\Omega(t)$ which maximizes the energy with a smooth constraint through a total variation term penalization obtained by :

$$\hat{\Omega} = \operatorname{argmax}_{\Omega} \int_{\mathbb{R}} |S_x(t, \Omega(t))|^2 dt - \lambda \int_{\mathbb{R}} \left| \frac{d\Omega}{dt}(t) \right|^2 dt, \quad (19)$$

where λ controls the importance of the smoothness of Ω .

When the ridges of several components have to be estimated, this method can be iterated after subtracting the energy located at the previously estimated ridge.

Audio source separation using AM/FM local estimation [Fourer et al. 2018]

Principle : We use TF local modulations to segregate independently evolving sources.

Assumptions : Sources are disjoint in the TF plane (each TF point is assigned to one source).

Mixture model

$$x(t) = \sum_{c=1}^C s_c(t) = \sum_{c=1}^C \left(\sum_{i_c \in \mathcal{I}_c} e^{\lambda_{i_c}(t) + j\phi_{i_c}(t)} \right) \quad (20)$$

$$\text{with } \lambda_x(t) = l_x + \mu_x t + \nu_x \frac{t^2}{2} \quad (21)$$

$$\text{and } \phi_x(t) = \varphi_x + \omega_x t + \alpha_x \frac{t^2}{2} \quad (22)$$

Coherent local modulation used for source separation

$$\text{CFM}_x(t, \omega) = \frac{\hat{\alpha}_x(t, \omega)}{\hat{\phi}_x(t, \omega)}, \quad \text{CAM}_x(t, \omega) = \frac{\hat{\lambda}_x(t, \omega)}{\hat{l}_x(t, \omega)}. \quad (23)$$

Results on real-world music signals (in terms of BssEval)

Table : Comparison of the voice/guitar separation results for the music piece AlexanderRoss VelvetCurtain [MedleyDB].

(a) New proposed estimator (ω_2)

Method	RQF (dB)	SIR (dB)	SDR (dB)	SAR (dB)
Oracle	8.70/9.66	19.02/23.21	8.08/9.24	8.50/9.44
CFM + k-means	5.88/6.02	9.77/11.14	4.58/4.84	6.58/6.32
CAM + k-means	2.47/2.62	2.57/11.70	0.23/1.49	0.84/ 9.96
CFM/CAM + k-means	2.99/3.16	3.65/ 12.16	0.63/1.93	1.20/8.36

(b) baseline modulation estimator (SM12)

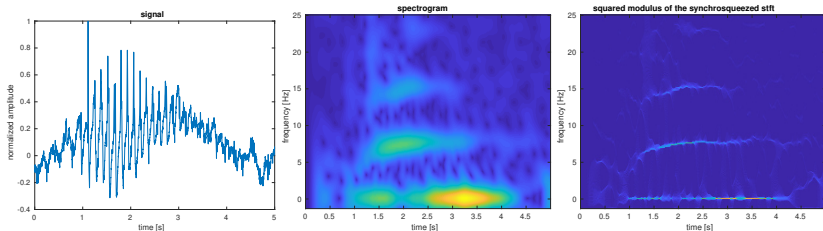
Method	RQF (dB)	SIR (dB)	SDR (dB)	SAR (dB)
Oracle	9.05/9.60	19.14/23.58	8.50/9.18	8.94/9.36
CFM + k-means	5.43/5.38	8.57/8.63	3.97/3.92	6.38/6.27
CAM + k-means	2.48/2.57	11.19/2.53	-0.14/1.33	0.50/9.41
CFM/CAM + k-means	2.38/2.47	11.53/2.46	-0.45/1.27	0.12/9.44

Audio examples at : <http://www.fourer.fr/publi/sp118>

Real-time HVS detection from EEG signals

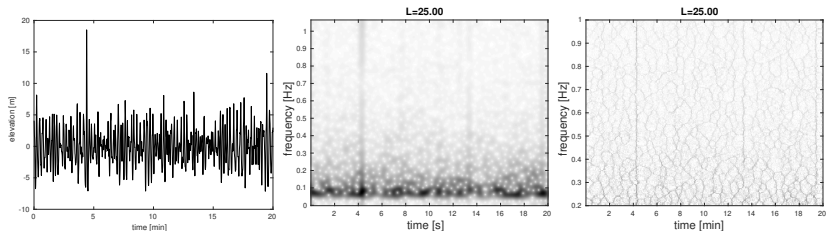
[Souriau, Fourer et al, 2019] (submitted at GRETSI 2019)

Principle : Combining recursive synchrosqueezing [Fourer et al. 2016] with a stochastic detection method.



Draupner wave analysis using time-reassigned synchrosqueezing

Principle : Introduce a new second-order time-reassigned synchrosqueezing method [Fourer, 2019] (submitted to GRETSI 19 and EUSIPCO 19)



The ANR PRC ASCETE project

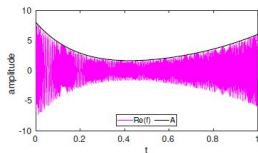


(with : LJK, ENS LYON, TelecomParisTech, LRI)

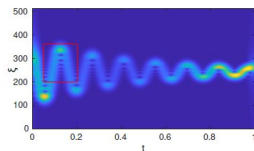
Goal : Combining deterministic and stochastic approaches for new signal processing methods

- Combining TF tools with machine learning (e.g. deep neural networks)
- Addressing difficult cases for mode recovery (e.g. overlapping components, noisy signals, etc.)
- Combining synchrosqueezing with Non-negative Matrix Factorization (NMF)
- Generalization to high dimension signals (images, tensors, etc.)
- New practical applications (perception, biomedicine, astronomy, etc.)

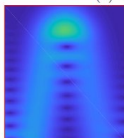
Improving the IF and chirp-rate estimation



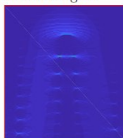
(a) Simulated signal



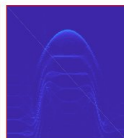
(b) STFT



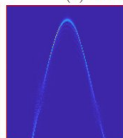
(c) STFT zoom



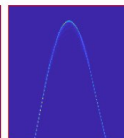
(d) FSST zoom



(e) FSST2 zoom



(f) FSST3 zoom



(g) FSST4 zoom

- Investigates new specific signal models (e.g. hyperbolic chirps, modulated impulses, etc.) [Pham et al. 2017] [Behera et al. 2016]
- Deals with overlapping components
- Considers stochastic signal models
- Further investigate signal and its transform properties (marginals, derivatives, reconstruction conditions, etc.) [Fourer et al 2017] [Fourer et al 2018]

Synchrosqueezing/Reassignment of new signal transforms

- Fully adaptive transform (Chirplet transform [Czarnecki,Fourer et al. 2018])
- High-order synchrosqueezing of the S-transform [Fourer, Auger, 2015],[Fourer, Auger, 2017]
- Discrete-time transforms (e.g. CQT, recursive CWT, etc.) [Fenet, Badeau, Richard 2017] [Fourer, Auger 2017]
- Investigate other transforms (with applications) : MDCT (data compression), scattering transform, Mellin (pattern recognition)

Future works perspectives

Proposals and open discussions

- Investigating deep neural networks (in particular CNNs)
- Investigating new applications : image processing, biology, signal processing on graphs.
- Reinforcing and reinterpreting methods with theory (ML and optimization).

Perspectives

- Collaboration with IRCAM (several works in progress related to neural networks).
- Collaboration with IREENA (several works in progress related to energy : NILM and smart grids).
- Collaboration with LaBRI (several works in progress related to perception : audio, speech, image, video processing).