High-Voltage Spindles detection from EEG signals using recursive synchrosqueezing transform

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Purpose of this work

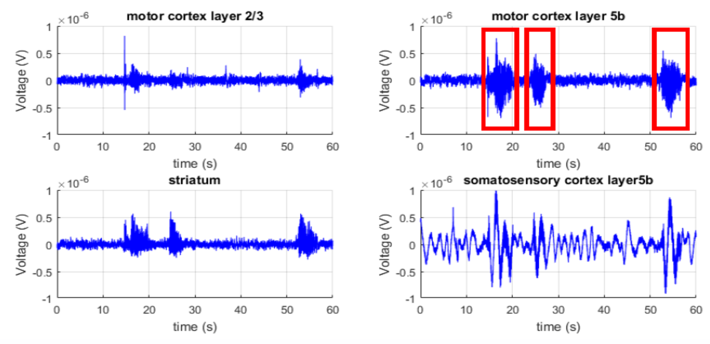
Goals :

- Detection of HVS (High-Voltage Spindles) signals from intracranial EEG for Parkinson disease control.
- Application on dataset of rat EEG.

Proposed approach :

- Apply a detector on a sharpened time-frequency representation.
- Use a fast implementation of the STFT based on recursive filtering to allow real-time applications.

Rat n°1



STFT definition and properties

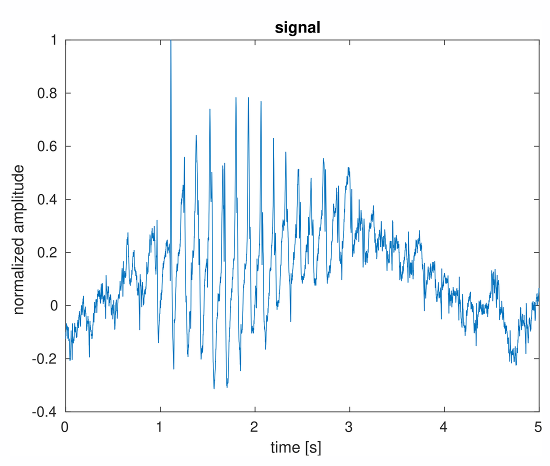
The STFT of signal $x(t)$ can be reworded as a convolution product with a filter $g(t,\omega) = h(t)\, e^{j\omega t}$ centered on frequency ω :

$$y_x^g(t,\omega) = \int_{-\infty}^{+\infty} g(\tau,\omega)\, x(t-\tau)\, d\tau = |y_x^g(t,\omega)|\, e^{j\Psi_x^g(t,\omega)} \quad (1)$$

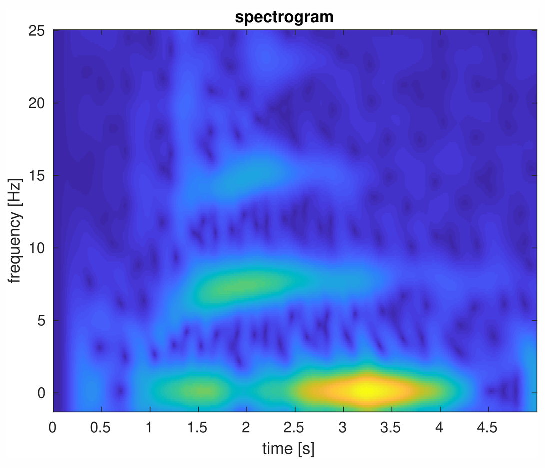
with $h(t)$ a real-valued analysis window, Ψ_x^g the phase.
$x(t)$ can be recovered from y_x^g with a time delay $t_0 \geq 0$ as :

$$x(t-t_0) = \frac{1}{h(t_0)} \int_{-\infty}^{+\infty} y_x^g(t,\omega)\, e^{-j\omega t_0} \frac{d\omega}{2\pi}, \quad (2)$$

when $\omega \mapsto y_x^g(t,\omega)$ is integrable and when $h(t_0) \neq 0$ (assumed to be true in the following).



(a) EEG signal $x(t)$ with HVS



(b) $|y_x^g(t,\omega)|^2$

Reassignment

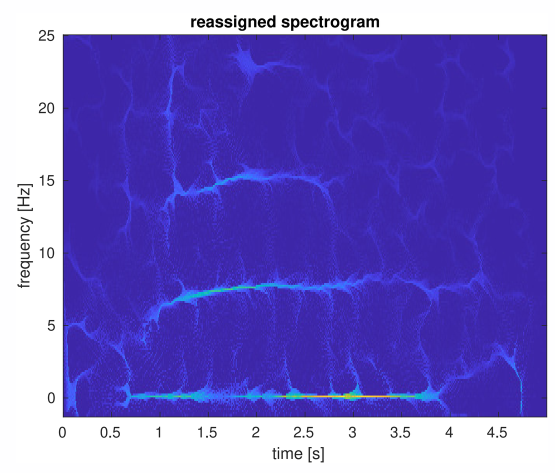
A sharpening technique [1] to improve the localization of the signal components. The reassignment operators are given by :

$$\hat{t}(t,\omega) = t - \frac{\partial \Psi_x^g}{\partial \omega}(t,\omega) \quad = t - \mathrm{Re}\left(\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right), \quad (3)$$

$$\hat{\omega}(t,\omega) = \frac{\partial \Psi_x^g}{\partial t}(t,\omega) \quad = \mathrm{Im}\left(\frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\right) \quad (4)$$

where $\mathcal{D}g(t,\omega) = \frac{\partial g(t,\omega)}{\partial t}$ and $\mathcal{T}g(t,\omega) = tg(t,\omega)$.
The reassigned spectrogram is computed as :

$$R_x^g(t,\omega) = \iint_{\mathbb{R}^2} |y_x^g(t',\omega')|^2 \delta(t - \hat{t}(t',\omega'))\, \delta(\omega - \hat{\omega}(t',\omega'))\, dt'\, d\omega' \quad (5)$$



(c) $R_x^g(t,\omega)$

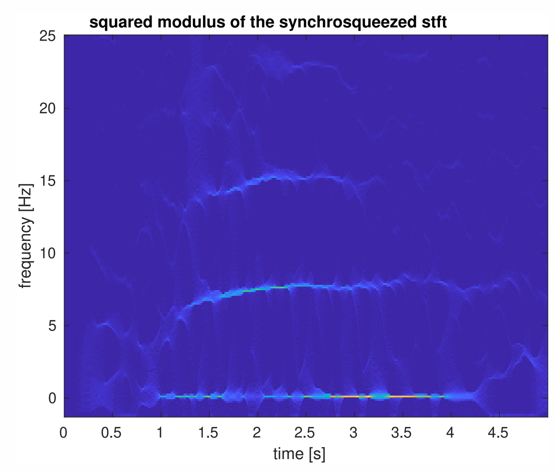
Synchrosqueezing

A variant of the reassignment method which admits a signal reconstruction formula [2]. The new transform can be deduced from the synthesis formula (2) as :

$$Sy_x^g(t,\omega) = \int_{\mathbb{R}} y_x^g(t,\omega')\, e^{-j\omega' t_0} \delta\left(\omega - \hat{\omega}(t,\omega')\right) d\omega'. \quad (6)$$

$x(t)$ can be recovered from the resulting TFR with a time-delay t_0 as :

$$\hat{x}(t-t_0) = \frac{1}{h(t_0)} \int_{\mathbb{R}} Sy_x^g(t,\omega) \frac{d\omega}{2\pi}. \quad (7)$$



(d) $|Sy_x^g(t,\omega)|^2$

Recursive implementation

A recursive implementation of y_x^g can be obtained if we use a causal recursive infinite impulse response filter [3] :

$$h_k(t) = \frac{t^{k-1}}{T^k (k-1)!}\, e^{-t/T}\, U(t), \quad (8)$$

$$g_k(t,\omega) = h_k(t)\, e^{j\omega t} = \frac{t^{k-1}}{T^k (k-1)!}\, e^{pt}\, U(t) \quad (9)$$

with $p = -\frac{1}{T} + j\omega$, $k \geq 1$ being the filter order, T the time spread of the window and $U(t)$ the Heaviside step function.
The impulse invariance method through the z-transform leads to the filter equation :

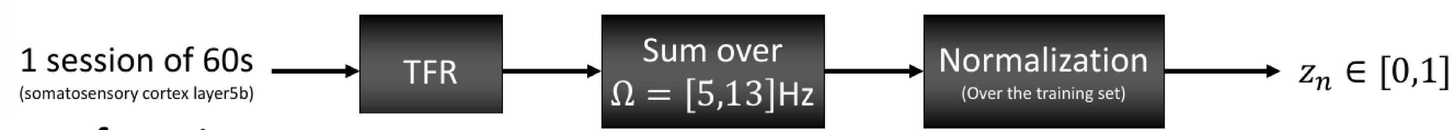
$$G_k(z,\omega) = T_s \mathcal{Z}\left\{g_k(t,\omega)\right\} = \frac{\sum_{i=0}^{k-1} b_i z^{-i}}{1 + \sum_{i=1}^{k} a_i z^{-i}}, \quad (10)$$

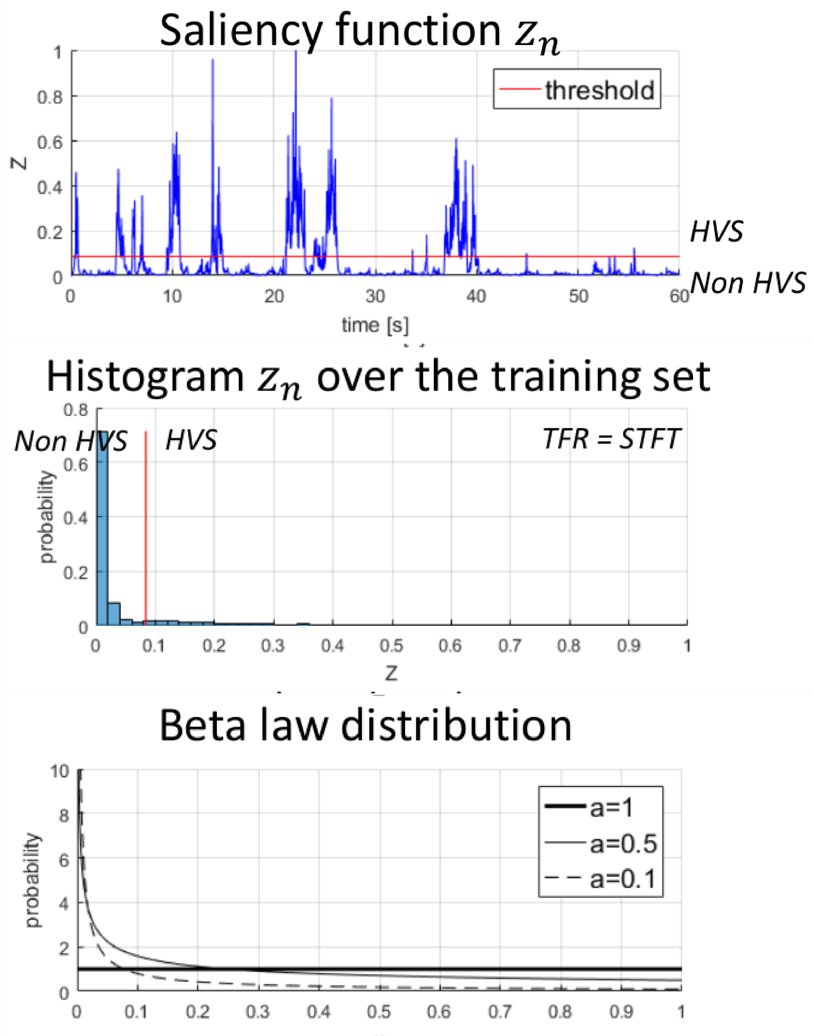
with T_s the sampling period. Thus, $y_k[n,m] \approx y_x^{g_k}(nT_s, \frac{2\pi m}{MT_s})$ can be computed from the sampled analyzed signal $x[n]$ using a standard recursive equation :

$$y_k[n,m] = \sum_{i=0}^{k-1} b_i\, x[n-i] - \sum_{i=1}^{k} a_i\, y_k[n-i,m] \quad (11)$$

with $n \in \mathbb{Z}$ and $m = 0, 1, \ldots, M-1$.

HVS detection based on control FDR





Decision rule

- H_0 : (HVS) the energy follows a uniform law : $f_0(z_n) = 1_{[0,1]}$
- H_1 : (non HVS) the energy follows a beta law : $f_1(z_n) = a z_n^{a-1} 1_{[0,1]}$.

Probability density function of z_n

$$f(z_n) = \pi_0 f_0(z_n) + (1-\pi_0) f_1(z_n) \quad (12)$$

Bayes factor

$$B_n = \frac{f_0(z_n)}{f_1(z_n)} = \frac{1}{a} z_n^{1-a} \underset{H_1}{\overset{H_0}{\gtrless}} 1 \quad (13)$$

Bayesian False Discovery Rate (FDR) for estimating a

$$bFDR = \Pr(H_0 | z_n \in W) = \frac{\pi_0 \Pr(z_n \in W | H_0)}{\Pr(z_n \in W)} = \pi_0 \frac{F_0(W)}{F(W)}. \quad (14)$$

$$\widehat{bFDR}(a) = \pi_0 \frac{N}{n_{r1}} a^{\frac{1}{1-a}}. \quad (15)$$

with W be the set of values associated to H_1 and n_{r1} its number of elements. We assume $\pi_0 = 1/2$.

HVS detection results

Each line in the Table corresponds to a rat.

- n_h denotes the number of detected HVS
- *thres* corresponds to the the (standardized) threshold estimated for each FDR.

The detection of HVS in EEG signal is a promising tool for the control of symptoms of the Parkinson's disease with a closed loop system. We test the synchrosqueezed STFT and the reassigned spectrogram for the detection of HVS and apply the automatic detection based on the control FDR.

As a reference, we compare our result with the method CWT+OTSU on one channel [4]. The results are expressed in terms of Sørensen-Dice score as :

$$\mathrm{DICE}(X,Y) = \frac{2|X \cap Y|}{|X| + |Y|}. \quad (16)$$

The recursively computed TFRs lead to a successful detection of the HVS with a lower delay than the ground truth [4].

n_h	FDR	synchrosqueezed STFT			reassigned spectrogram		
		thres	delay [s]	dice	thresh	delay [s]	dice
19	1%	0,013	0,282	0,645	0,006	-0,220	0,615
	2%	0,029	0,415	0,558	0,025	0,011	0,847
	5%	0,082	0,554	0,301	0,076	0,220	0,581
6	1%	0,008	0,007	0,614	0,008	-0,403	0,612
	2%	0,024	0,151	0,721	0,023	-0,125	0,780
	5%	0,071	0,598	0,614	0,070	0,3710	0,698
7	1%	0,007	-0,011	0,450	0,006	-0,436	0,327
	2%	0,019	0,168	0,703	0,017	-0,154	0,700
	5%	0,063	0,298	0,606	0,060	0,052	0,794
3	1%	0,017	0,146	0,829	0,018	0,019	0,960
	2%	0,036	0,334	0,838	0,037	0,146	0,921
	5%	0,093	0,488	0,838	0,095	0,567	0,604
2	1%	0,002	-0,073	0,505	0,001	-0,181	0,319
	2%	0,007	0,050	0,845	0,007	-0,220	0,849
	5%	0,026	0,261	0,926	0,024	-0,068	0,986
2	1%	0,016	0,214	0,9620	0,016	-0,172	0,966
	2%	0,033	0,36	0,967	0,032	-0,018	0,988
	5%	0,083	0,728	0,952	0,082	0,192	0,979
1	1%	0,018	0,242	0,867	0,017	-0,115	0,951
	2%	0,036	0,267	0,872	0,036	0,035	0,981
	5%	0,090	0,399	0,893	0,092	0,067	0,964

Conclusions and future work

- A new detection method for HVS detection from EEG signals based on the synchrosqueezing transform
- Allows real-time implementation thanks to a recursive filtering implementation.
- **Future work will investigate the signal detection model and the parameters of the computed TFRs to improve our detection results.**

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