The ASTRES Toolbox for Mode Extraction of Non-Stationary Multicomponent Signals

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Abstract

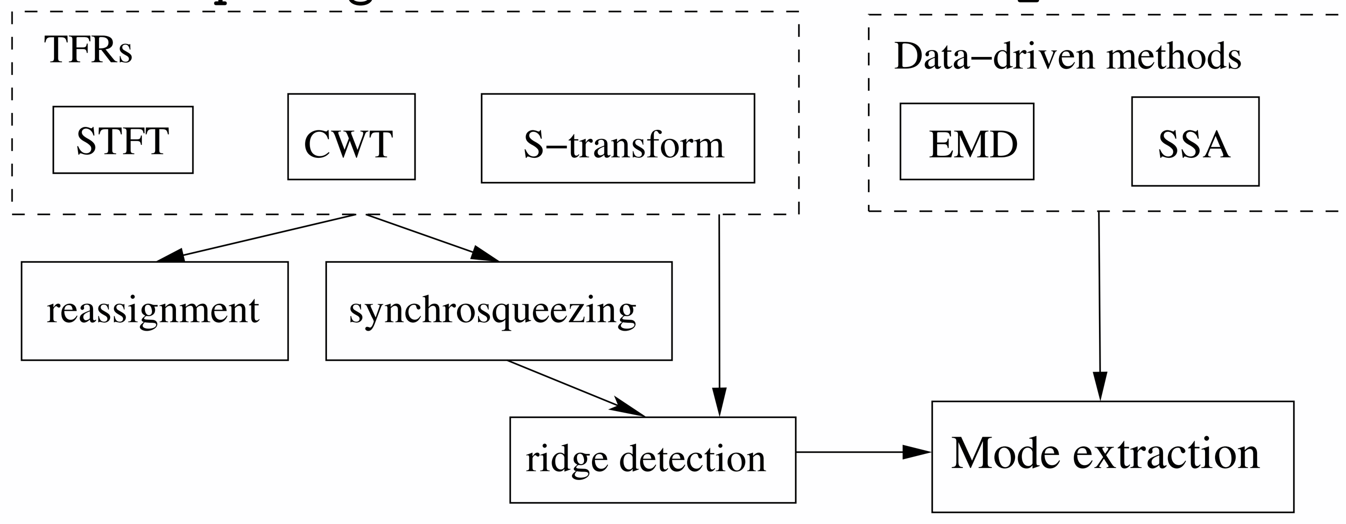
The French project called ASTRES (Analysis-Synthesis-Transformation by Reassignment, EMD and Synchrosqueezing) project and its related toolbox aim at offering advanced tools designed for processing non-stationary and multicomponent signals. The goal of this toolbox is to share with the scientific community Matlab implementations of new (or very recent) methods for analysis, synthesis and transformation of any signal made of physically meaningful components (e.g. sinusoids, trends or noise). The proposed techniques contain several of our recent contributions which are now unified into the same framework and strengthened from a theoretical point of view. They can provide efficient time-frequency or time-scale representations and allow elementary components extraction.

Each proposed method is numerically illustrated on real-world signals:

- **the gravitational wave signal Livingston GW150914,**
- **a multicomponent audio signal of a recorded cello.**

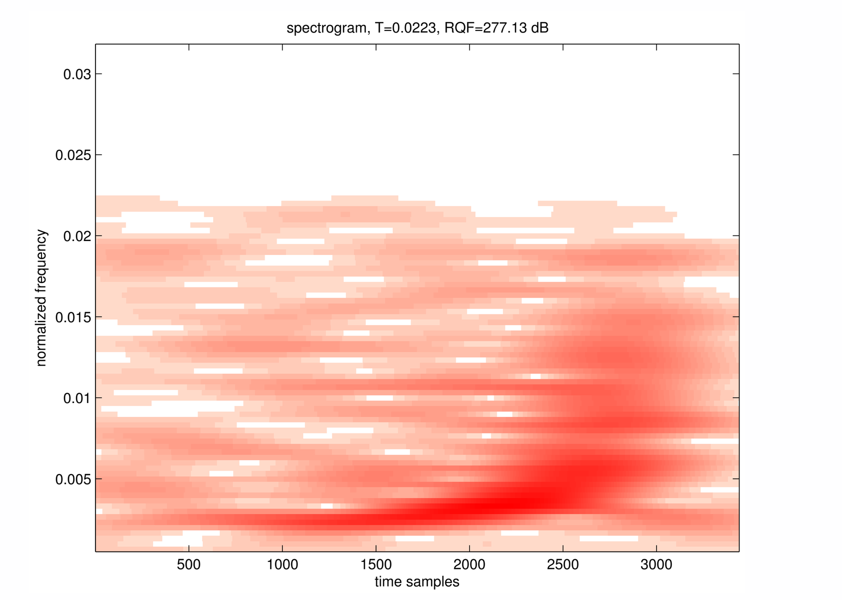
Overview of the toolbox

`https://github.com/dfourer/ASTRES_toolbox`



Short-Time Fourier Transform (STFT)

STFT and spectrogram



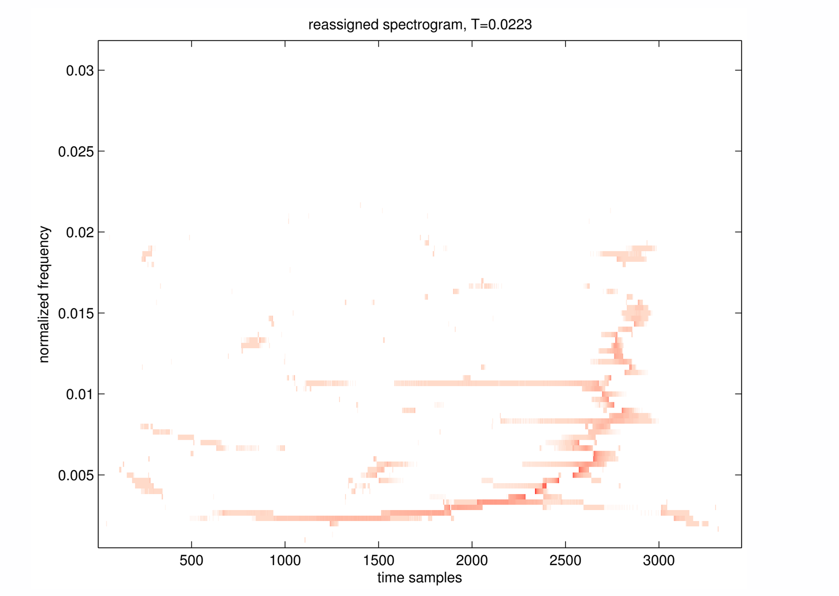
$|F_x^h(t,\omega)|^2$

STFT of a signal x using a differentiable analysis window h:

$$F_x^h(t,\omega) = \int_{\mathbb{R}} x(u)h(t-u)^* \, \mathbf{e}^{-j\omega u}\, \mathrm{d}u.$$

- Allows a recursive implementation using a specific analysis window [1]: $h_k(t) = \frac{t^{k-1}}{T^k(k-1)!}\,\mathbf{e}^{-t/T}\,U(t)$, $k \geq 1$, $U(t)$ being the Heaviside step function and T a time spread parameter.

Reassigned spectrogram

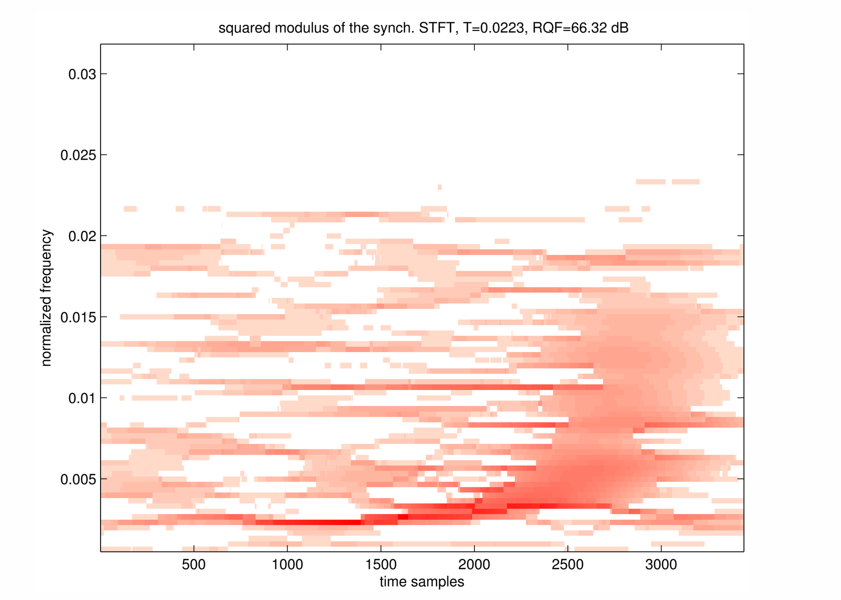


$\mathrm{RF}_x(t,\omega)$

Provides a sharpen (reassigned) time-frequency representation (TFR) thanks to the reassignment operators (introduced by Kodera *et al.* in 1976 and generalized by Auger et Flandrin in 1995).

- The resulting TFR is **not reversible**.
- The time-frequency localization can be adjusted through a damping parameter using the Levenberg-Marquardt algorithm [1]

Synchrosqueezed STFT

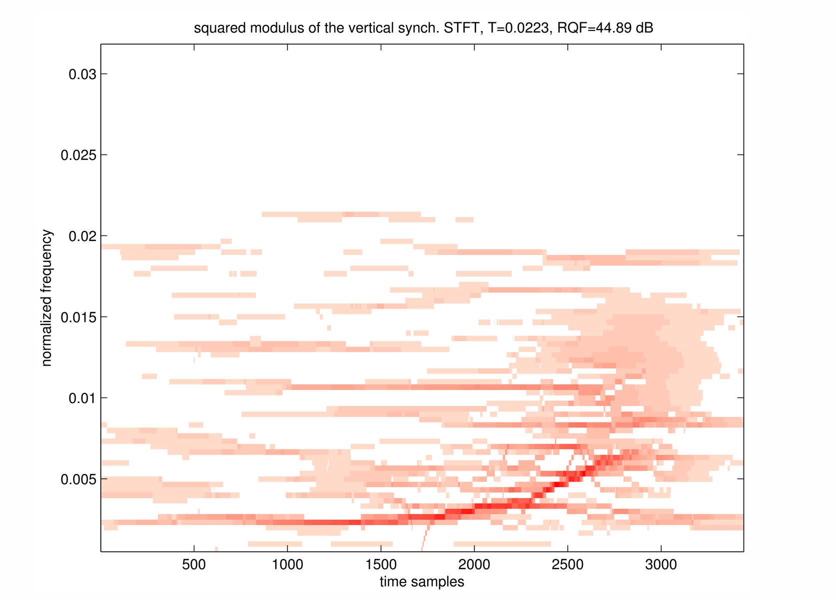


$|\mathrm{SF}_x(t,\omega)|^2$

Provides thanks to the frequency reassignment operator, a sharpen (with a poorer localization than reassignment) TFR which admits a signal reconstruction formula.

- The resulting TFR is **reversible and allows mode extraction**.
- Can also be recursively implemented for real-time computation [1].
- The time-frequency localization can be adjusted through a damping parameter using Levenberg-Marquardt algorithm [1].

Vertically synchrosqueezed STFT



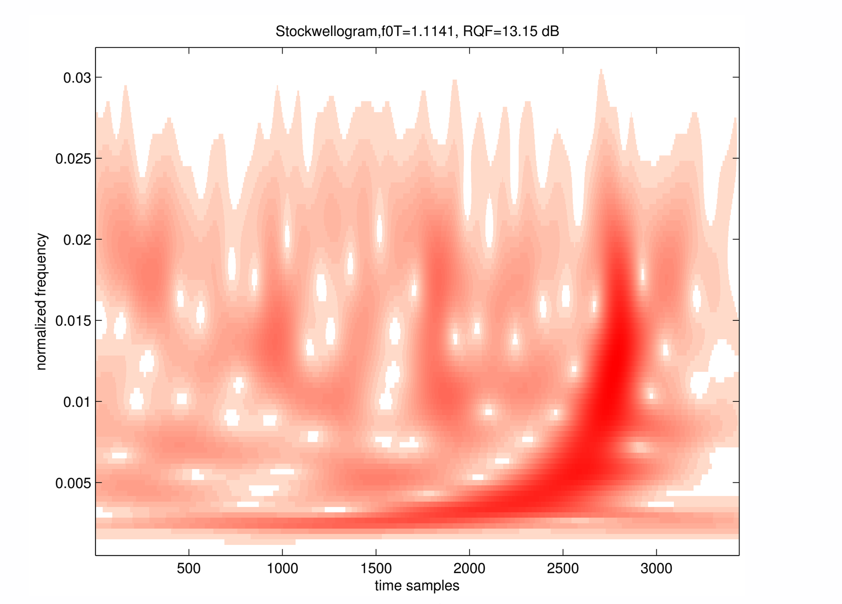
$|\mathrm{VSF}_x(t,\omega)|^2$

Improvement of the localization of the synchrosqueezing method thanks to an enhanced instantaneous frequency estimator [4], [5].

- The resulting TFR is **reversible and allows mode extraction**.
- Better frequency localization than using the classical synchrosqueezed STFT.

Continuous Wavelet Transform (CWT) and S-Transform (ST)

CWT/ST and scalogram/Stockwellogram



$|\mathrm{ST}_x(t,\omega)|^2$

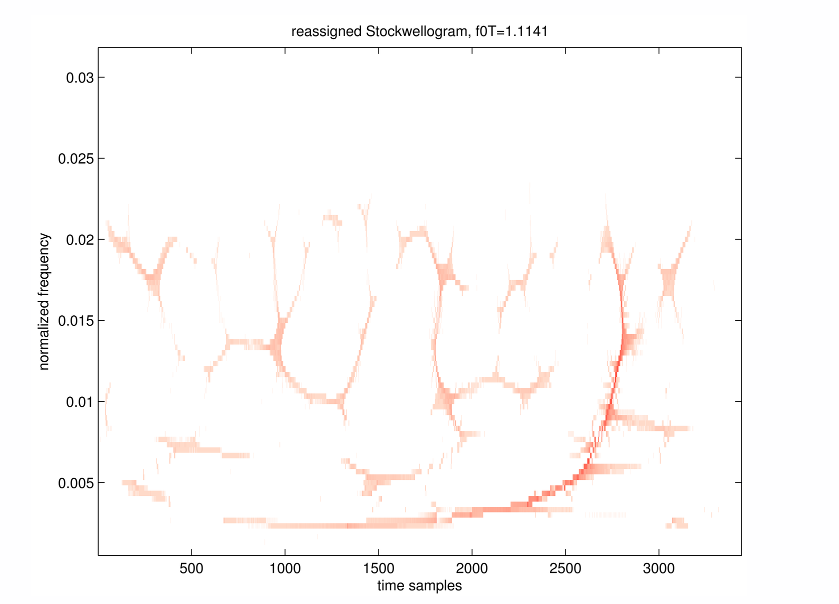
Using the Morlet mother wavelet $\Psi(t) = \frac{\pi^{-1/4}}{\sqrt{T}}\,\mathbf{e}^{\frac{-t^2}{2T^2}}\,\mathbf{e}^{j\omega_0 t}$, the CWT is expressed as

$$\mathrm{MW}_x(t,\omega) = \sqrt{\frac{|\omega|}{\omega_0 T\sqrt{\pi}}}\int_{-\infty}^{+\infty} x(\tau)\,\mathbf{e}^{-\frac{\omega^2(t-\tau)^2}{2(\omega_0 T)^2}}\,\mathbf{e}^{-j\omega(\tau-t)}\,\mathrm{d}\tau$$

and the S-transform can now be defined as:

$$\mathrm{ST}_x(t,\omega) = \sqrt{\frac{|\omega|}{2\sqrt{\pi}\omega_0 T}}\,\mathbf{e}^{-j\omega t}\,\mathrm{MW}_x(t,\omega).$$

Reassigned scalogram/Stockwellogram

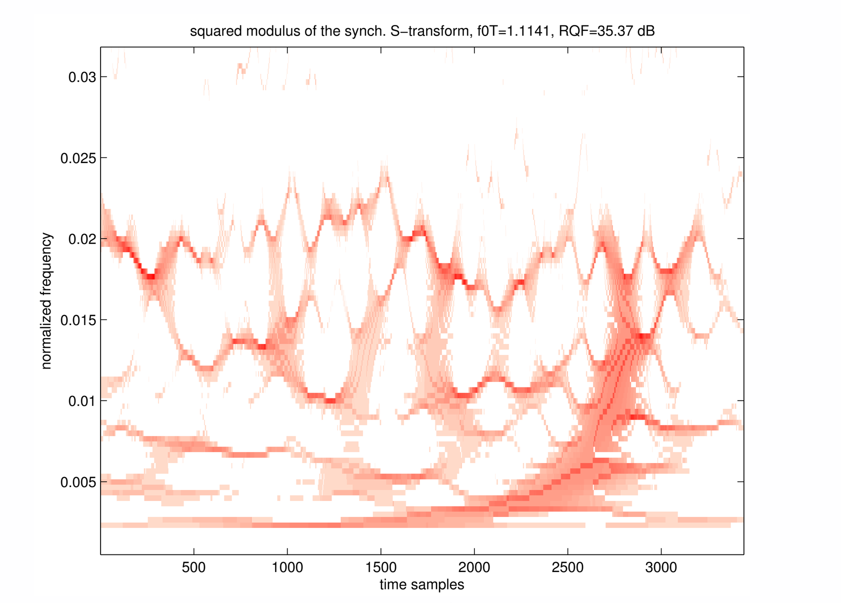


$\mathrm{RST}_x(t,\omega)$

Provides a sharpen (reassigned) time-frequency representation (TFR) thanks to dedicated reassignment operators (with an elegant formulation expressed in terms of the original transform using modified mother wavelet functions).

- The resulting TFR **is not reversible**.
- The time-frequency localization can now also be adjusted through a damping parameter (as for the STFT) using the Levenberg-Marquardt algorithm [2] [3].
- Both CWT and its reassigned scalogram, can be recursively computed (for real-time applications) using a specific mother wavelet function [2].

Synchrosqueezed CWT/ST

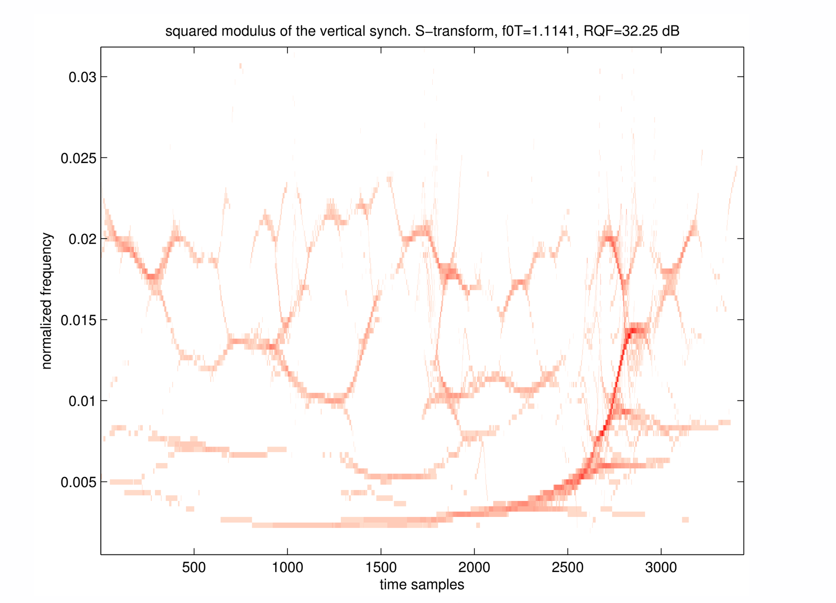


$|\mathrm{SST}_x(t,\omega)|^2$

As for the STFT, synchrosqueezing provides a sharpen TFR which admits a signal reconstruction formula.

- The resulting TFR (defined as the squared modulus of the synchrosqueezed transform) is **reversible and allows mode extraction**.
- The localization can also be adjusted through a damping parameter [2],[3].

Vertically synchrosqueezed CWT/ST



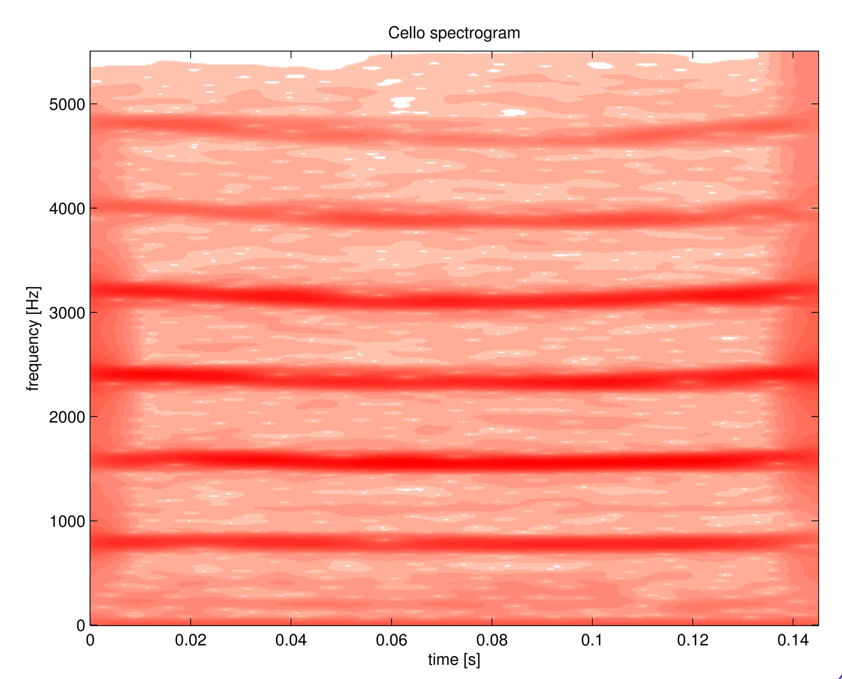
$|\mathrm{VSST}_x(t,\omega)|^2$

Localization improvement of both the synchrosqueezed CWT [6] and the synchrosqueezed ST [3] thanks to a dedicated enhanced instantaneous frequency estimator.

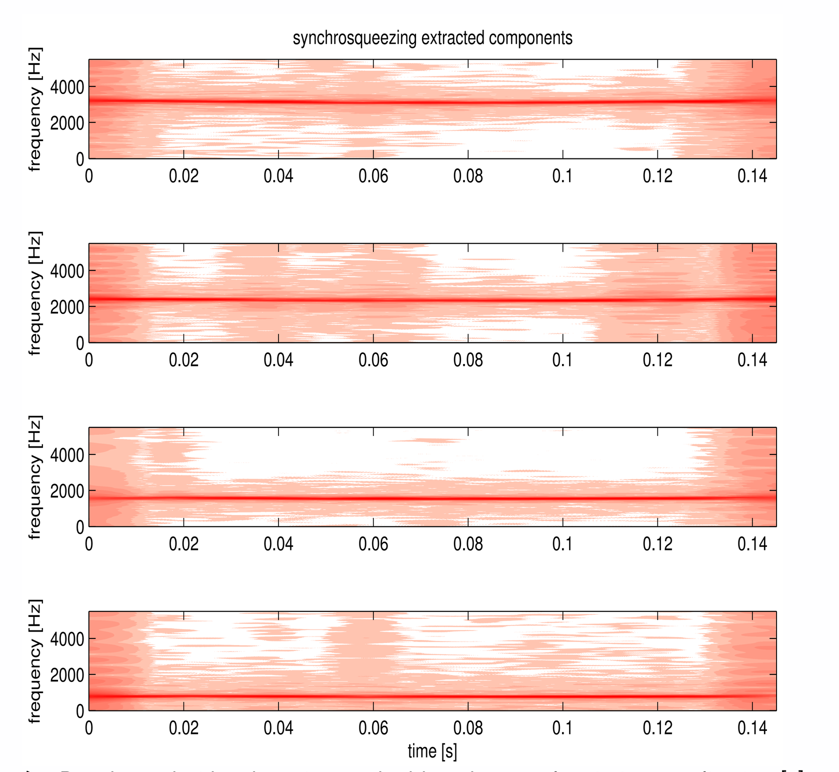
- The resulting TFR (defined as the squared modulus of the synchrosqueezed transform) is **reversible and allows mode extraction**.
- Better frequency localization than the classical synchrosqueezed CWT/ST.

Data-driven methods and ridge detection for mode extraction

reference signal

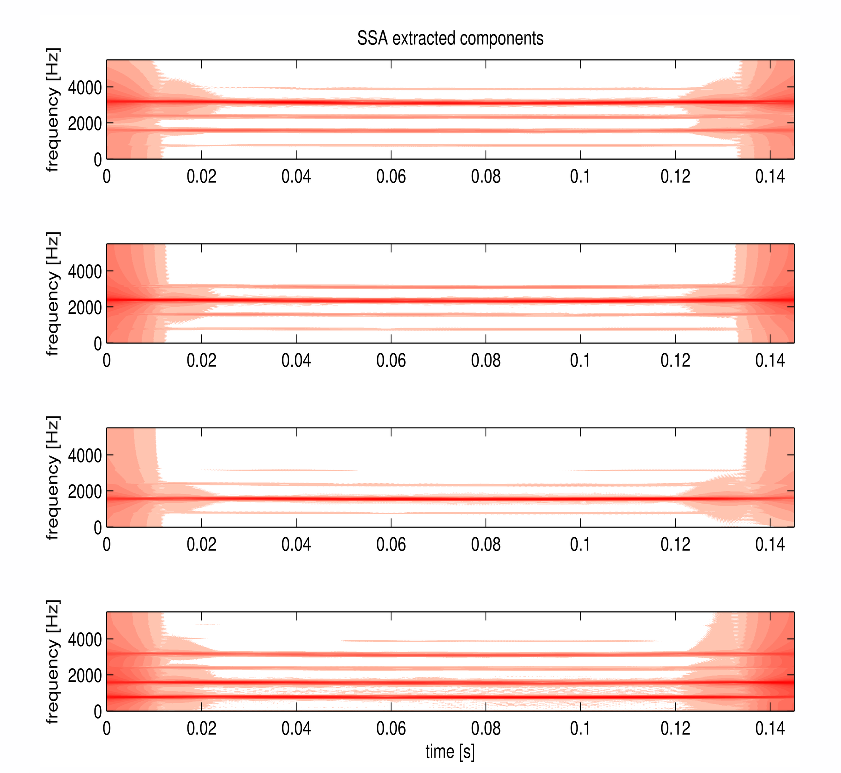


Synchrosqueezing + ridge-detection



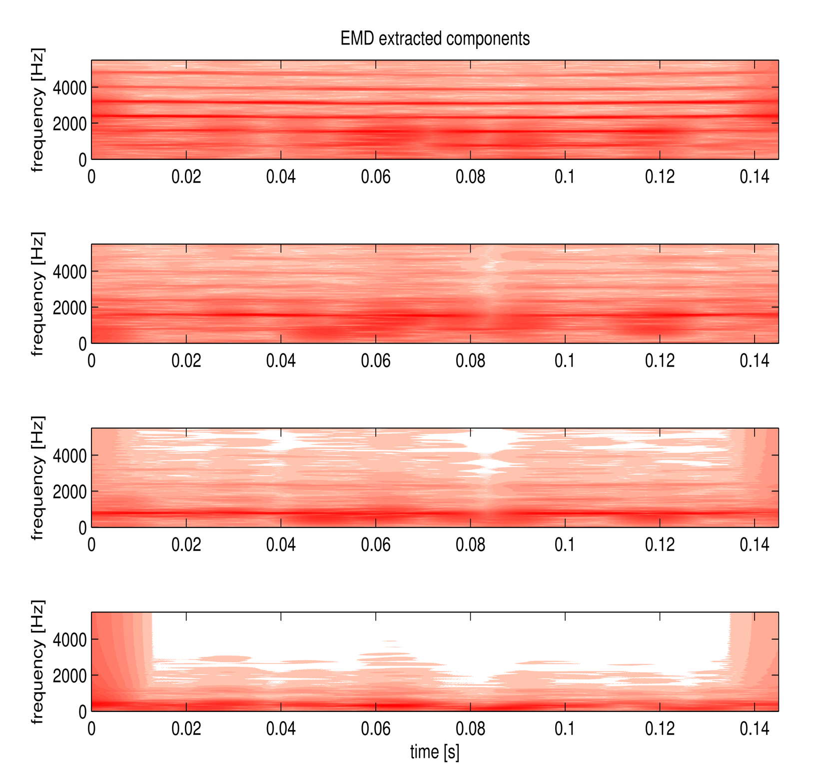
- *Brevdo et al.* ridge detection method based on total variation penalization [5].
- *Flandrin* method for ridge detection based on spectrogram zeros filtering [6].

Singular Spectrum Analysis (SSA)



- A new proposed fully automatic SSA method for components extraction [9].
- A new method (recently submitted for publication) called sliding SSA.

Empirical Mode Decomposition (EMD)



Several recent contributions such as [10] for a theoretical reformulation of the EMD algorithm and an extension for 2D signals [11].

Conclusion and future work

The French project called ASTRES toolbox was introduced as a collection of Matlab functions for processing non-stationary and multicomponent signals. This toolbox unifies into the same framework several recent techniques developed into the ASTRES project. Some methods are designed for efficient TFRs computation and mode extraction, were used to provide new results on real world signals. Future work consists in theoretically strengthening these tools, and new practical applications.

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