High-Voltage Spindles detection from EEG signals using recursive synchrosqueezing transform

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Résumé – Cet article propose une nouvelle méthode de détection appliquée sur des signaux d'électroencéphalographie de rats parkinsoniens lors d'une expérimentation animale. Notre technique repose sur une implémentation par bancs de filtres récursifs de la transformée de Fourier à court terme que nous améliorons par l'utilisation de plusieurs méthodes de réallocation temps-fréquence. Un détecteur est ensuite appliqué sur la représentation obtenue afin d'identifier l'apparition de signaux HVS qui caractérisent les sujets atteints de maladies neurodégénératives. Nos résultats montrent que cette approche améliore les performances de l'état de l'art tout en permettant un traitement temps réel.

Abstract – This paper proposes a new detection technique applied on electroencephalogram (EEG) signals which were measured on Parkinsonian rats during an experiment. Our technique uses a recursive filter-bank-based implementation of the short-time Fourier transform that is sharpened using several time-frequency reassignment methods. A detector is then applied to the obtained representation to allow an identification of the HVS signals which are specific to subjects with neurodegeneratives diseases. Our results show an improvement of the state of the art while paving the way of a real-time implementation.

1 Introduction

Biosignals such as electroencephalogram (EEG), electromyogram (EMG) or electrocorticogram (EcoG) can be viewed as a mixture of several time-varying components. Such signals can be addressed by time-frequency analysis [1] which offers an efficient framework to deal with non-stationary components while allowing to disentangle them [2]. Short-Time Fourier Transform (STFT) and Continuous Wavelet Transform (CWT) belong to the most popular methods for which a significant number of implementations were realized (*e.g.* audio application, radar signal processing, seismic, etc.).

Today, the contributions in time-frequency analysis aim at improving the accuracy and the robustness of a computed Time-Frequency Representation (TFR) to fulfill the requirement of an arbitrary specific application. For instance, the Reassignment (RST) [3] and the Synchrosqueezing Transform (SST) [4] were introduced to efficiently improve the readability of a TFR leading to new denoising or empirical mode extraction applications [2, 1]. Among these contributions, a recursive filter-based implementation of the synchrosqueezing transform was proposed for the STFT [5] and the CWT in [6] paving the way for novel real-time applications.

In the present study, we propose to combine the recursive SST with a detection method to identify particular patterns in EEG signal. Excessive beta oscillations (called β-waves henceforth) were found to be associated with spatial memory impairments [7]. Silencing High-Voltage-Spindle (HVS) with deep brain stimulation (DBS) that resembles to the pathophysiological β-waves is expected to delay the development of β-waves. Thus, it may also delay the progression of Parkinson Disease (PD) motor symptoms that are observed during the waking immobility of patients. Despite rat signals are investigated in this paper, our approach remains full of interest for preventing the crisis of Parkinsonian patients.

Our paper is organized as follows : Section 2 describes the recursive implementation of the reassigned and synchrosqueezed STFT ; an algorithm based on the Benjamini-Hochberg procedure is presented in Section 3 for (almost) real-time HVS detection in TFRs ; then preliminary results are given in Section 4 before concluding the paper in Section 5 with future works directions.

2 Recursive reassignment and synchrosqueezed STFT

2.1 Definitions and properties

The STFT of a time signal $x(t)$ is a function of the time t and of the frequency ω that can be defined as a linear convolution product between the analyzed signal and the complex-valued impulse response of a bandpass filter $g(t,\omega)=h(t)\,\mathrm{e}^{j\omega t}$ centered on frequency ω :

$$y_x^g(t,\omega) = \int_{-\infty}^{+\infty} g(\tau,\omega)x(t-\tau)\,d\tau = |y_x^g(t,\omega)|\,\mathrm{e}^{j\Psi_x^g(t,\omega)} \quad (1)$$

where $h(t)$ is a real-valued analysis window, Ψ_x^g is the phase and j the complex number such as $j^2 = -1$. The resulting TFR is the spectrogram defined as $|y_x^g(t,\omega)|^2$. Thus, the original signal $x(t)$ can be recovered from y_x^g with a time delay $t_0 \geq 0$ using the following synthesis formula :

$$x(t-t_0) = \frac{1}{h(t_0)} \int_{-\infty}^{+\infty} y_x^g(t,\omega)\, \mathrm{e}^{-j\omega t_0}\, \frac{d\omega}{2\pi}, \tag{2}$$

when $\omega \mapsto y_x^g(t,\omega)$ is integrable and when $h(t_0) \neq 0$ (assumed to be true in the following).

2.2 Reassignment

The reassignment method [3] is a sharpening technique which can be applied on a TFR to improve the localization of the signal components by reassigning the values of an energy distribution to time-frequency coordinates that are closer to the real support of the analyzed signal. The reassignment operators of the spectrogram can be related to the phase of the STFT and computed as a function of STFT involving particular filter functions [5] :

$$\hat{t}(t,\omega) = t - \frac{\partial \Psi_x^g}{\partial \omega}(t,\omega) \qquad = t - \mathrm{Re}\left(\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right), \tag{3}$$

$$\hat{\omega}(t,\omega) = \frac{\partial \Psi_x^g}{\partial t}(t,\omega) \qquad = \mathrm{Im}\left(\frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\right) \tag{4}$$

where $\mathcal{D}g(t,\omega) = \frac{\partial g(t,\omega)}{\partial t}$ and $\mathcal{T}g(t,\omega) = tg(t,\omega)$. Finally, the reassigned spectrogram is computed as :

$$R_x^g(t,\omega) = \iint_{\mathbb{R}^2} |y_x^g(t',\omega')|^2 \delta(t - \hat{t}(t',\omega'))\delta(\omega - \hat{\omega}(t',\omega'))\, dt'\, d\omega' \tag{5}$$

where $\delta(t)$ denotes the Dirac distribution. This TFR is sharpened but unfortunately non-reversible due to the loss of the phase information.

2.3 Synchrosqueezed STFT

The synchrosqueezing method [8] is a variant of the reassignment method which also provides a sharpened linear time-frequency transform but now admits a signal reconstruction formula. Its consists in moving the transform instead of its energy according to the frequency reassignment operator. The new transform can be deduced from the synthesis formula (2) as :

$$\mathrm{Sy}_x^g(t,\omega) = \int_{\mathbb{R}} y_x^g(t,\omega')\, \mathrm{e}^{-j\omega' t_0}\, \delta\left(\omega - \hat{\omega}(t,\omega')\right)\, d\omega'. \tag{6}$$

As a result, $|\mathrm{Sy}_x^g(t,\omega)|^2$ provides a sharpened TFR and $x(t)$ can be estimated with a time-delay t_0 by :

$$\hat{x}(t-t_0) = \frac{1}{h(t_0)} \int_{\mathbb{R}} \mathrm{Sy}_x^g(t,\omega)\, \frac{d\omega}{2\pi}. \tag{7}$$

The signal components can be disentangled and individually recovered as proposed in [2] by restricting the integration area to the vicinity of each ridge.

2.4 Recursive implementation

A recursive implementation of y_x^g can be obtained if we use for $h(t)$ a causal recursive infinite impulse response filter [5] :

$$h_k(t) = \frac{t^{k-1}}{T^k (k-1)!}\, \mathrm{e}^{-t/T}\, U(t), \tag{8}$$

$$g_k(t,\omega) = h_k(t)\, \mathrm{e}^{j\omega t} = \frac{t^{k-1}}{T^k (k-1)!}\, \mathrm{e}^{pt}\, U(t) \tag{9}$$

with $p = -\frac{1}{T} + j\omega$, $k \geq 1$ being the filter order, T the time spread of the window and $U(t)$ the Heaviside step function.

Hence, the impulse invariance method leads to the following formulation of the filter defined by Eq. (9) :

$$G_k(z,\omega) = T_s \mathcal{Z}\left\{g_k(t,\omega)\right\} = \frac{\sum_{i=0}^{k-1} b_i z^{-i}}{1 + \sum_{i=1}^{k} a_i z^{-i}}, \tag{10}$$

with $b_i = \frac{1}{L^k (k-1)!} B_{k-1,k-i-1} \alpha^i$, $\alpha = \mathrm{e}^{pT_s}$, $L = T/T_s$, $\mathcal{Z}\left\{f(t)\right\} = \sum_{n=0}^{+\infty} f(nT_s) z^{-n}$, $a_i = A_{k,i}\left(-\alpha\right)^i$, T_s being the sampling period. $B_{k,i} = \sum_{j=0}^{i} (-1)^j A_{k+1,j} (i+1-j)^k$ denotes the Eulerian numbers and $A_{k,i} = \binom{k}{i} = \frac{k!}{i!(k-i)!}$ the binomial coefficients.

Thus, $y_k[n,m] \approx y_x^{g_k}(nT_s, \frac{2\pi m}{MT_s})$ can be computed from the sampled analyzed signal $x[n]$ using a standard recursive equation :

$$y_k[n,m] = \sum_{i=0}^{k-1} b_i\, x[n-i] - \sum_{i=1}^{k} a_i\, y_k[n-i,m] \tag{11}$$

where $n \in \mathbb{Z}$ and $m = 0, 1, ..., M-1$ are respectively the discrete time and frequency indices. The z-transform of the other specific impulse responses can be expressed as functions of $G_k(z,\omega)$ at different orders :

$$T_s \mathcal{Z}\{\mathcal{T} g_k(t,\omega)\} = kT G_{k+1}(z,\omega) \tag{12}$$

$$T_s \mathcal{Z}\left\{\mathcal{D} g_k(t,\omega)\right\} = \frac{1}{T} G_{k-1}(z,\omega) + p G_k(z,\omega) \tag{13}$$

which hold for any $k \geq 1$ provided that $G_0(z,\omega) = G_{-1}(z,\omega) = 0$. As a result, the reassignment operators can be computed as :

$$\hat{n}[n,m] = n - \mathrm{Round}\left(\mathrm{Re}\left(\frac{T_s^{-1} y_x^{\mathcal{T}g}[n,m]}{y_x^g[n,m]}\right)\right) \tag{14}$$

$$\hat{m}[n,m] = \mathrm{Round}\left(\frac{M}{2\pi} \mathrm{Im}\left(\frac{T_s y_x^{\mathcal{D}g}[n,m]}{y_x^g[n,m]}\right)\right) \tag{15}$$

and both the reassignment and synchrosqueezing methods can respectively be implemented [1] according to Eqs. (5) and (6).

1. Matlab© full implementation of the code freely available at : `https://github.com/dfourer/ASTRES_toolbox`

3 HVS detection based on control FDR

We focus on the presence of energy located in the expected frequency range of a HVS : $\Omega = [5, 13]$ Hz. Hence, a simple detector can consider the following saliency function with values in $[0, 1]$ as proposed in [9] :

$$z_n = \begin{cases} \frac{1}{K} \sum_{m\in\Omega} \mathrm{TFR}[n,m] & \text{, if } \sum_{m\in\Omega} \mathrm{TFR}[n,m] < K \\ 1 & \text{otherwise} \end{cases} \tag{16}$$

where $\mathrm{TFR}[n,m]$ can arbitrary be replaced by the spectrogram $|y_x^h[n,m]|^2$, the reassigned spectrogram $R_x^g[n,m]$ or the squared modulus of the synchrosqueezed STFT $|Sy_x^h[n,m]|^2$. In practice, the normalization factor $K = \max_n(\sum_{m\in\Omega} \mathrm{TFR}[n,m])$ is estimated using one minute of observation of a training dataset. The probability density function of z_n can be modeled as a mixture of a beta distribution and a uniform distribution with different parameters as illustrated in Fig. 2. Now, we assume that we have a finite number N of pairs (z_n, h_n) where h_n denotes the latent unit to predict. Among the samples, most of them are assumed to follow the null hypothesis H_0 (non HVS) while the others follow the alternative hypothesis H_1 (HVS) :

- H_0 : the energy follows a uniform law : $f_0(z_n) = 1_{[0,1]}$
- H_1 : the energy follows a beta law : $f_1(z_n) = a z_n^{a-1} 1_{[0,1]}$.

Thus, the observed mixture density can be expressed as :

$$f(z_n) = \pi_0 f_0(z_n) + (1 - \pi_0) f_1(z_n) \tag{17}$$

with $\pi_0 = \Pr(H_0)$. In order to reject or not H_0, we compute the Bayes factor B_n such as :

$$B_n = \frac{f_0(z_n)}{f_1(z_n)} = \frac{1}{a} z_n^{1-a} \underset{H_1}{\overset{H_0}{\gtrless}} 1 \tag{18}$$

where parameter a can be estimated using the false discovery control on multiple tests [10]. Let W be the set of values associated to H_1 (non HVS). Thus, the Bayesian False Discovery Rate (FDR) can be expressed as :

$$bFDR = \Pr(H_0|z_n \in W) = \frac{\pi_0 \Pr(z_n \in W|H_0)}{\Pr(z_n \in W)} = \pi_0 \frac{F_0(W)}{F(W)}. \tag{19}$$

Let W_{r1} be the set of the samples in the reject region with n_{r1} its number of elements. The $bFDR$ can be computed as :

$$bFDR = \pi_0 \frac{F_0(W_{r1})}{F(W_{r1})} = \pi_0 \frac{a^{\frac{1}{1-a}}}{F(W_{r1})} \tag{20}$$

where $F(W_{r1})$ can be approximated using the empirical cumulative function : $\widehat{F}(W_{r_1}) = \frac{n_{r1}}{N}$. We finally get the following estimator of the Bayesian FDR :

$$\widehat{bFDR}(a) = \pi_0 \frac{N}{n_{r1}} a^{\frac{1}{1-a}}. \tag{21}$$

Assuming $\pi_0 = 1/2$, we use the following estimator for a to compute the detection threshold :

$$\widehat{a} = \underset{a}{\operatorname{argmin}} |\widehat{bFDR}(a) - FDR|. \tag{22}$$

4 Numerical results

4.1 Dataset description

The PD rat models were induced in 3-4 months old Sprague-Dawley rats by unilateral injection of 6-OHDA in the medial forebrain bundle (MFB). To this end, an integrated electrophysiology instrument suitable for a DBS procedure was used. The rats were unilaterally implanted by bipolar stimulation using an electrode into the ipsilateral sub-thalamus nuclei (STN). The electrode was lowered slowly along the dorsal ventral axis of the brain and then advanced ventrally to the STN to obtain an electrophysiological signal sampled at $F_s = 1000$ Hz with a strikingly silent structure. The EEG signals were collected from a group of 34 lesioned rats and 20 control rats in both sleeping and waking immobile states in order to study the behaviour of spindling and non-spindling characteristics. The prominent frequency components of HVS are around 5-13Hz and the duration of each HVS episode is from 1 to 4 seconds.

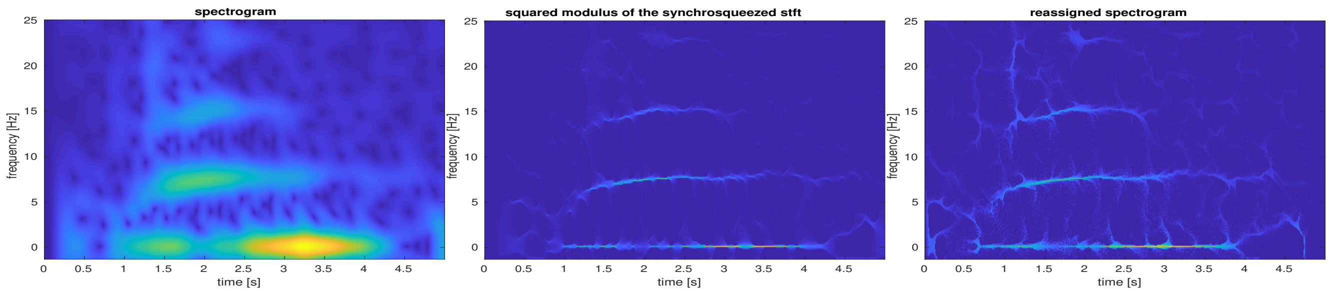
Example TFRs of an EEG signal containing a HVS are displayed in Fig. 1 : the recursive spectrogram in Fig.1(a), the recursive squared modulus of the synchrosqueezed STFT in Fig.1(b) and the reassigned spectrogram in Fig.1(c). These TFRs are computed using $k = 5$, $M = 8000$ and $L = 5$ and the analyzed frequencies are limited to the $[0, 25]$ Hz range. The synchrosqueezed STFT is computed with a delay $n_0 = (k - 1)L$ which corresponds to the maximum of the analysis window $h_k[n]$. Our results clearly illustrate the sharpening capability of the reassignment and of the synchrosqueezing methods when applied on an EEG signal in comparison to the STFT. The main advantage of our proposal is that each TFR can be computed in real-time thanks to the recursive filtering implementation provided by Eq. (11).

4.2 Detection results

Our results use the squared modulus of the recursive synchrosqueezed STFT and the reassigned spectrogram which are considered as the input TFRs of the saliency function (16). Before estimating the delay of detection, we apply a median filter (of range 1 second) to reduce the effect of the noise. The used detection threshold is trained on the first 60 seconds of each analyzed signal. Table 1 presents the resulting Sørensen-dice coefficients of each experiment which are defined as : $\mathrm{DICE} = (|H_0 \cap \widehat{H_0}|)/(|H_0| + |\widehat{H_0}|)$ where the “ground truth” reference is defined in [11]. Thus, we obtain excellent detection results (with high dice values), especially using the recursive reassigned spectrogram. Moreover, the resulting negative delays show that our proposal can detect HVS earlier (*i.e.* faster) than the reference thanks to the time-reassignment operation and despite the resulting slightly lower dice coefficients.

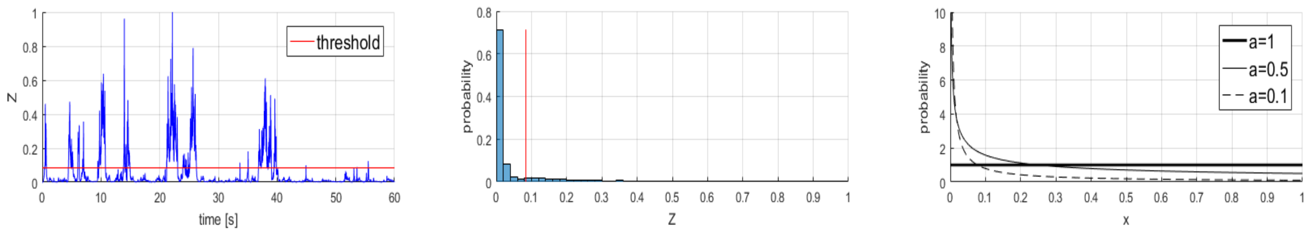
5 Conclusion and future works

We have proposed a new HVS detection method based on the recursive implementation of the reassigned and synchrosqueezed STFT. Our results show that the recursively computed TFRs lead to a successful detection of the HVS with a lower de-



(a) spectrogram (b) synchrosqueezed STFT (c) reassigned spectrogram

FIGURE 1 – Comparisons of the different recursive TFRs computed for an EEG signal with a HVS.



(a) saliency function (b) histogram (c) beta law distribution

FIGURE 2 – Resulting saliency function (a) and its histogram (b) computed from the recursive synchrosqueezed STFT of an EEG signal. Its probability density function can be compared to a beta distribution (c) with parameter $a \in \{0.1, 0.5, 1\}$.

TABLE 1 – HVS detection results provided by the proposed method. Each line corresponds to a PD rat. n_h denotes the number of detected HVS and $thres$ corresponds to the the (standardized) threshold estimated for each FDR.

n_h	FDR	synchrosqueezed STFT			reassigned spectrogram		
		thres	delay [s]	dice	thresh	delay [s]	dice
19	1%	0,013	0,282	0,645	0,006	-0,220	0,615
	2%	0,029	0,415	0,558	0,025	0,011	0,847
	5%	0,082	0,554	0,301	0,076	0,220	0,581
6	1%	0,008	0,007	0,614	0,008	-0,403	0,612
	2%	0,024	0,151	0,721	0,023	-0,125	0,780
	5%	0,071	0,598	0,614	0,070	0,3710	0,698
7	1%	0,007	-0,011	0,450	0,006	-0,436	0,327
	2%	0,019	0,168	0,703	0,017	-0,154	0,700
	5%	0,063	0,298	0,606	0,060	0,052	0,794
3	1%	0,017	0,146	0,829	0,018	0,019	0,960
	2%	0,036	0,334	0,838	0,037	0,146	0,921
	5%	0,093	0,488	0,838	0,095	0,567	0,604
2	1%	0,002	-0,073	0,505	0,001	-0,181	0,319
	2%	0,007	0,050	0,845	0,007	-0,220	0,849
	5%	0,026	0,261	0,926	0,024	-0,068	0,986
2	1%	0,016	0,214	0,9620	0,016	-0,172	0,966
	2%	0,033	0,36	0,967	0,032	-0,018	0,988
	5%	0,083	0,728	0,952	0,082	0,192	0,979
1	1%	0,018	0,242	0,867	0,017	-0,115	0,951
	2%	0,036	0,267	0,872	0,036	0,035	0,981
	5%	0,090	0,399	0,893	0,092	0,067	0,964

lay than using classical methods. For a small FDR control, we also successfully detect HVS before the ground truth using the reassigned spectrogram. Future works will further investigate the signal detection model and the parameters of the computed TFRs to improve our detection results.

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