## Purpose of this work

### Goals:
- Detection of HVS (High-Voltage Spindles) signals from intracranial EEG for Parkinson disease control.
- Application on a dataset of rat EEG.

### Proposed approach:
- Apply a detector on a sharpened time-frequency representation.
- Use a fast implementation of the STFT based on recursive filtering to allow real-time applications.

## STFT definition and properties

The STFT of signal $x(t)$ can be written as a convolution product with a filter $g(t,-\omega)e^{-j\omega t}$ centered on frequency $\omega$:

$$y(t,\omega) = \int_{-\infty}^{\infty} g(t,\tau)e^{-j\omega \tau}d\tau = y(t,\omega)|_{\tau=t}$$

with $h(t)$ a real-valued analysis window, $y(t,\omega)$ the phase $x(t)$ can be recovered from $y(t,\omega)$ with a time delay $\tau_0 \geq 0$ as:

$$x(t-\tau_0) = \frac{1}{\pi^2} \int_{\omega=-\infty}^{\infty} y(t,\omega)e^{-j\omega \tau_0}d\omega$$

when $\omega \rightarrow y(t,\omega)$ is integrable and when $h(t_0) \neq 0$ (assumed to be true).

## Reassignment

A sharpening technique [1] to improve the localization of the signal components. The reassignment operators are given by:

$$\hat{r}(t,\omega) = t - \frac{\partial}{\partial \omega} \arg \left( \frac{Y(t,\omega)}{\sqrt{Y(t,\omega)}^2 + \delta^2} \right)$$

where $\delta$ is the interference parameter. The reassignment of the STFT is computed as:

$$\hat{X}(t,\omega) = \int_{\omega = \omega_0}^{\omega_1} \hat{r}(t,\omega)|_{\omega_0}^{\omega_1}d\omega$$

## Recursive implementation

A recursive implementation of the STFT can be obtained if we use a causal recursive infinite impulse response filter [3]:

$$h_k(t) = \frac{1}{\pi^2} \int_{\omega = \omega_0}^{\omega_1} \frac{Y(t,\omega)}{\sqrt{Y(t,\omega)}^2 + \delta^2} \frac{e^{-j\omega t}}{\omega + j\delta} d\omega$$

with $\omega \rightarrow y(t,\omega)$ is integrable and when $h(t_0) \neq 0$ (assumed to be true).

## HVS detection based on control FDR

Each line in the Table corresponds to a rat.
- $n_0$ denotes the number of detected HVS.
- $n_3$ corresponds to the threshold (standardized) threshold estimated for each FDR.

The detection of HVS in EEG signal is a promising tool for the control of the symptoms of Parkinson’s disease with a closed loop system. We test the synchrocosqueezed STFT and the reassignment spectrum of the STFT for detection and apply the automatic detection based on the control FDR.

As a reference, we compute our result with the method CWT+OTSU on one channel [4]. The results are expressed in terms of Sørensen–Dice score as:

$$\text{DICE}(X, Y) = \frac{2 \times X \cap Y}{|X| + |Y|}$$

The recursively computed TFs lead to a successful detection of the HVS with a lower delay than the ground truth [4].

## Synchrocosqueezing

A variant of the reassignment method which assigns a signal reconstruction formula [2]. The new transform can be defined from the synthesis formula (2) as:

$$\hat{X}(t,\omega) = \int_{\omega = \omega_0}^{\omega_1} \frac{Y(t,\omega)}{\sqrt{Y(t,\omega)}^2 + \delta^2} \frac{e^{-j\omega t}}{\omega + j\delta} d\omega$$

## Decision rule

- $H_0$ (HVS) the energy follows a uniform law $f_\omega(z) = \frac{1}{\pi}$
- $H_1$ (non-HVS) the energy follows a beta law $f_\omega(z) = \frac{z^{a-1}}{\Gamma(a)}$,

With $\Gamma(a)$ the Gamma function.

### Probability density function of $z_i$

$$f(z_i) = const(z_i^a + (1 - z_i)^{a-1})$$

### Bayesian False Discovery Rate (FDR) for estimating $a$

$$FDR = Pr(H_0|z_i) = \frac{\int W(z_i)}{Pr(H_0)} = \frac{\int W(z_i)}{\int W(z_i)}$$

### Bayesian False Discovery Rate

$$bFDR = bPr(H_0|z_i) = \frac{\int W(z_i)}{\int W(z_i)}$$

## Conclusions and future work

- A new detection method for HVS detection from EEG signals based on the synchrocosqueezing transform.
- Allows real-time implementation thanks to a recursive filtering implementation.
- Future work will investigate the signal detection model and the parameters of the computed TFs to improve our detection results.

## Bibliography