Estimation of Instantaneous Frequency and Amplitude of Multi-Component Signals using Sparse Modeling of Signal Innovation

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***Abstract*—This paper introduces a new method for estimating modes in non-stationary mixture signals. First, we establish a connection between the short-time Fourier transform (STFT) and sparse sampling theory, representing observations as pulses filtered by a known function. Leveraging the finite rate of innovation in the target signal, our specialized reconstruction approach enables mode estimation amidst noise. Second, we propose a variant based on a recursive version of the STFT allowing real-time mode parameter estimation with sequential acquisition. We compare our results with state-of-the-art methods, showing an improvement in estimation performance across various scenarios. Our approach paves the way of the future mode disentangling algorithms based on Finite rate of innovation.**

***Index Terms*—Ridge extraction, Time-frequency, Sparse deconvolution, Finite rate of innovation.**

I. Introduction

Complex signals from various physical systems are often modeled as Multi Component Signal (MCS), represented as a sum of amplitude- and frequency-modulated (AMFM) sine waves. Separating and estimating the parameters of individual components or modes in an MCS are essential in various applications [1], [2]. Existing approaches, such as Empirical Mode Decomposition (EMD) [3] and Singular Spectrum Analysis (SSA) [4], as well as methods projecting the signal onto the time-frequency (TF) plane [5], aim to achieve this. Linear Time-Frequency Representation (TFR), like Short-Time Fourier Transform (STFT) or Continuous Wavelet Transform (CWT), are extensively studied for their ability to provide a framework where the Instantaneous Frequency (IF) trajectory of each mode corresponds to a ridge in the TF plane, enabling applications such as noise removal [6] or source separation [7]. The robustness of such methods to noise underscores the importance of regularization in ridge detection [8]. Synchrosqueezing-based method with Total Variation (TV) regularization [9], [10], and spline interpolation [11] address IF estimation challenges. However, most of the existing methods either do not allow for the estimation of other parameters such as Instantaneous Amplitude (IA) or involve considerable computational time [12]. Usually, such methods require the entire signal spectrogram and have limitations for possible real-time parameter estimation through sequential acquisition.

In this context, we propose a new method for IF and IA estimation through spectrogram analysis. Connecting with sparse sampling theory [13], [14], we emphasize that, in noiseless scenarios, each instantaneous slice of the spectrogram can be accurately modeled as a Stream of Dirac (SoD) considered as an ideal TFR [10]. Thus, we address the problem of estimating IF and IA to identify the position and weight of each Dirac pulse in the signal exhibiting a Finite Rate of Innovation (FRI) in the presence of undesired noise [13], [15]. To make our approach more robust, we resort to an alternative to the Prony method: the Total Least-Squares (TLS) approach [15]. Moreover, we introduce a recursive filter-bank-based implementation using specific analysis windows designed for real-time and adaptive applications. Additionally, we consider vertical SynchroSqueezing Transform (SST) [10] as an alternative sharpened representation for comparison. The main contributions of the paper can be summarized as follows:

- A novel IF and IA estimation methods based on a sparse observation model of MCS spectrograms.
- A FRI-based method possibly combined with SST for disentangling and reconstructing the modes.
- A recursive filter-bank-based implementation using a specific causal analysis window.

This paper is organized as follows. In Section II, we introduce the problem addressed in this work and our proposed observation model. Section III and IV present respectively the IF and IA reconstruction strategy. We introduce a recursive implementation in Section V enabling future real-time mode retrieval applications. The performance of the proposed method is comparatively assessed in Section VI through numerical experiments. Conclusions and future work are finally reported in Section VII.

II. Observation model

Let x be a discrete-time finite-length mixture signal made of K superimposed Amplitude- and Frequency-Modulated

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(AM-FM) components expressed as:

$$x(n) = \sum_{k=0}^{K-1} x_k(n) = \sum_{k=0}^{K-1} \alpha_k(n)\, \mathrm{e}^{j\frac{2\pi\phi_k(n)}{M}} \tag{1}$$

where $n \in \{0, 1, \ldots, N-1\}$ denotes the time instant, $j^2 = -1$, $\alpha_k(n) \geq 0$ and $\phi_k(n)$ denote respectively the time-varying amplitude and phase of the k-th component. Here, K is assumed to be either known or estimated [16]. In this study, we focus on estimating ridge positions and amplitudes from the observed mixture of TFR. They are specifically associated with the IF $\phi'_k(n)$, the sampled derivative with respect to time of ϕ_k, and the IA $\alpha_k(n)$ of each component. Now, we consider the STFT of x, using an analysis window θ that can be defined at each TF point as:

$$F_x^\theta(n,m) = \sum_{l=-\infty}^{+\infty} x(l)\theta(n-l)^*\, \mathrm{e}^{-j\frac{2\pi lm}{M}} \tag{2}$$

with $m \in \{0, 1, \ldots, M-1\}$ and z^* being the complex conjugate of z. Let $\boldsymbol{S}$ be an $\mathbb{R}^{M\times N}$ matrix representing the spectrogram of x. After disregarding noise, assuming only non-modulated sinusoidal components, and neglecting the interferences between close or overlapping components, $\boldsymbol{S}$ can be approximated as follows [17], [18]:

$$[\boldsymbol{S}]_{n,m} = |F_x^\theta(n,m)|^2 \approx \sum_{k=0}^{K-1} |x_k(n)|^2 \left|F_\theta\left(m - \phi'_k(n)\right)\right|^2 \tag{3}$$

where $F_\theta(m) = \frac{1}{M}\sum_{l\in\mathbb{Z}} \theta(l)\, \mathrm{e}^{-j\frac{2\pi lm}{M}}$ is the discrete Fourier transform of θ. In the sequel, we denote the nth spectrogram column as $[\boldsymbol{S}]_{n,:}$ that is represented by $\boldsymbol{s}_n = [s_{n,0}, \ldots, s_{n,M-1}]^\top \in \mathbb{R}^M$ that is modeled according to Eq. (3) as:

$$s_{n,m} := \sum_{k=0}^{K-1} (\alpha_k(n))^2 g(m - \phi'_k(n)) \tag{4}$$

with $g(m) = |F_\theta(m)|^2$, and where $\alpha_k(n)$ and $\phi'_k(n)$ are the components IF and IA to estimate. Therefore, Equation (4) can be interpreted as a field of Dirac Pulses (DPs) situated on the ridge of each component $f_n(m) = \sum_{k=0}^{K-1}(\alpha_k(n))^2\delta(\frac{m-\phi'_k(n)}{M})$, where the Dirac distribution is convolved with a known kernel g. Hence, our work consists in estimating each DP position and weight from $\boldsymbol{s}_n$.

III. Instantaneous Frequency (IF) estimation

The restoration of sparse signals in that context was intensively studied over the past few years [13], [15], [19]. The locations and weights of the DPs can be retrieved using the Fourier series coefficients of f_n [13]. From Eq. (4) we obtain:

$$\begin{aligned} s_{n,m} &= \sum_{k=0}^{K-1} (\alpha_k(n))^2 \sum_{\lambda=-\infty}^{\infty} F_g(\lambda)\, \mathrm{e}^{j\frac{2\pi\lambda(m-\phi'_k(n))}{M}} \\ &= \sum_{\lambda=-\infty}^{\infty} F_g(\lambda) \underbrace{\sum_{k=0}^{K-1} (\alpha_k(n))^2\, \mathrm{e}^{-j\frac{2\pi\lambda\phi'_k(n)}{M}}}_{F_{f_n}(\lambda)}\, \mathrm{e}^{j\frac{2\pi\lambda m}{M}}. \end{aligned} \tag{5}$$

In order to avoid the use of an infinite sum, a bandlimited approximation can be used in Eq. (5) such that only $2M_0+1$ Fourier series coefficients are kept [14]:

$$s_{n,m} \approx \sum_{\lambda=-M_0}^{M_0} F_g(\lambda) F_{f_n}(\lambda)\, \mathrm{e}^{j\frac{2\pi\lambda m}{M}} \tag{6}$$

which rewrites matrix-wise with the Fourier series coefficients $F_{f_n}(\lambda), \lambda \in [-M_0, M_0]$ of f_n such as $\digamma_n = [F_{f_n}(-M_0), F_{f_n}(-M_0+1), \cdots, F_{f_n}(M_0)]^T$ as:

$$\boldsymbol{s}_n = \boldsymbol{V}\boldsymbol{D}_g \digamma_n \Leftrightarrow \digamma_n = \boldsymbol{D}_g^{-1}\boldsymbol{V}^\dagger \boldsymbol{s}_n \tag{7}$$

where $[\boldsymbol{V}]_{m,\lambda} = \mathrm{e}^{j\frac{2\pi m\lambda}{M}}$ is a $M\times(2M_0+1)$ matrix, $\boldsymbol{V}^\dagger$ is the pseudo-inverse (eg. $(\boldsymbol{V}^T\boldsymbol{V})^{-1}\boldsymbol{V}^T$) of $\boldsymbol{V}$ and $\boldsymbol{D}_g$ is a diagonal matrix gathering the discrete Fourier series coefficients of g in $[-M_0, M_0]$. In noiseless setting, $\phi'_k(n)$ can be estimated using the Prony method [20], which computes a filter $\boldsymbol{h} = [h(1), h(2), \cdots, h(K)]^T$ that annihilates F_{f_n}, thus we have:

$$(F_{f_n} \star \boldsymbol{h})(l) = \sum_{k=0}^{K-1} (\alpha_k(n))^2\, \mathrm{e}^{-j\frac{2\pi l\phi'_k(n)}{M}} H\left(\mathrm{e}^{-j\frac{2\pi\phi'_k(n)}{M}}\right) = 0 \tag{8}$$

with $\star$ the convolution product operator, and $H(z) = \sum_{i\in\mathbb{Z}} h(i)z^{-i}$ the z-transform of $\boldsymbol{h}$ whose roots are $\mathrm{e}^{-j\frac{2\pi\phi'_k(n)}{M}}$. Assuming $h(0) = 1$, [15], [21], $\boldsymbol{h}$ can be retrieved by solving a linear Yule-Walker system which has a unique solution if $M_0 \geq 2K$:

$$\underbrace{\begin{pmatrix} F_{f_n}(0) & \cdots & F_{f_n}(-K+1) \\ F_{f_n}(1) & \cdots & F_{f_n}(-K+2) \\ \vdots & \ddots & \vdots \\ F_{f_n}(K-1) & \cdots & F_{f_n}(0) \end{pmatrix}}_{\boldsymbol{A}} \begin{pmatrix} h(1) \\ h(2) \\ \vdots \\ h(K) \end{pmatrix} = - \begin{pmatrix} F_{f_n}(1) \\ F_{f_n}(2) \\ \vdots \\ F_{f_n}(K) \end{pmatrix} \tag{9}$$

where the $\phi'_k(n)$ of each component is deduced from the roots of $\boldsymbol{h}$. Different alternative to the Prony method have been proposed to perform the estimation in the presence of noise [22], [23]. From them, the TLS approach [15] has the advantage of being simple and computationally attractive, while circumventing the ill-posed problem in Eq. (9) by approximating $\boldsymbol{h}$. It consists in finding the filter that minimizes $\|\boldsymbol{A}\boldsymbol{h}\|^2$, subject to the constraint $\|\boldsymbol{h}\|^2 = 1$, instead of the perfect annihilation of F_{f_n}. This minimization of $\|\boldsymbol{A}\boldsymbol{h}\|^2$ is a known problem that can be solved by computing the Singular Values Decomposition (SVD) of $\boldsymbol{A}$ [15], and by setting $\boldsymbol{h}$ to the eigenvector associated with the smallest eigenvalue. The sensitivity to the choice of M_0 in the presence of noise has been studied in [14], [15] and is thus not discussed here.

IV. Instantaneous Amplitude (IA) estimation

The classical principle of FRI reconstruction enables IA estimation once the IF values are determined. Indeed, considering the definition of F_{f_n} given in Eq. (5), the amplitudes can be obtained by solving a linear system of equations with a unique solution:

$$\begin{pmatrix} W_{0,0} & \cdots & W_{0,K-1} \\ W_{1,0} & \cdots & W_{1,K-1} \\ \vdots & \ddots & \vdots \\ W_{K-1,0} & \cdots & W_{K-1,K-1} \end{pmatrix} \begin{pmatrix} (\alpha_0(n))^2 \\ (\alpha_1(n))^2 \\ \vdots \\ (\alpha_{K-1}(n))^2 \end{pmatrix} = \begin{pmatrix} F_{f_n}(0) \\ F_{f_n}(1) \\ \vdots \\ F_{f_n}(K-1) \end{pmatrix} \quad (10)$$

where $W_{l,k} = e^{-j\frac{2\pi l \phi'_k(n)}{M}}$. Due to its sensitivity to outliers, it is generally preferred [14], [15] to address a least squares problem which leads to the following estimator:

$$\hat{\alpha}_k^{(LS)}(n) = \underset{\alpha_k(n)}{\operatorname{argmin}} \sum_{|\lambda| \leq M_0} \left| F_{f_n}(\lambda) - \sum_{k=0}^{K-1} (\alpha_k(n))^2 \, \mathbf{e}^{-j\frac{2\pi\lambda\phi'_k(n)}{M}} \right|^2 . \quad (11)$$

However, even with perfectly estimated $\phi'_k(n)$, this method still exhibits inefficiency, particularly in the presence of outliers and when component frequency modulations are neglected. Thus, we also propose another simpler amplitude estimator based on the properties of the STFT [24]:

$$\hat{\alpha}_k(n) = \left| \frac{F_x^\theta(n, m_k)}{F_\theta(m_k - \phi'_k(n))} \right| \quad (12)$$

where $m_k \in \{0, 1, \cdots, M-1\}$ is the nearest integer frequency bin (on the grid) from $\phi'_k(n)$.

V. Recursive implementation

We propose to extend our work by the use of a recursive filtering implementation to allow real-time and filter-bank-based signal processing applications. To this end, we resort to the use of a specific causal analysis window related to an Infinite Impulse Response (IIR) filter [25]:

$$\theta_p(n) = \frac{n^{p-1}}{L^p(p-1)!} \, \mathbf{e}^{-n/L} U(n) \quad (13)$$

where p is the order of the related recursive filter, L is a parameter which controls the spread of the analysis window and $U(n)$ is the Heaviside function. The Fourier transform of this filter is expressed as $F_{\theta_p}(m) = (1 + j\frac{2\pi m L}{M})^{-p}$ which leads to [25]:

$$g(m) = |F_{\theta_p}(m)|^2 = \left(1 + \left(\frac{2\pi m L}{M}\right)^2\right)^{-p} \quad (14)$$

that is used in the FRI observation model given in Eq. (4). The proposed analysis window in Eq. (13) allows the computation of $F_x^{\theta_p}(n,m) = y_p(n,m) \, \mathbf{e}^{j\frac{2\pi n m}{M}}$ where y_p is the filtered signal computed through a standard recursive equation:

$$y_p(n,m) = \sum_{i=0}^{p-1} b_i \, x(n-i) - \sum_{i=1}^{p} a_i \, y_p(n-i, m) \quad (15)$$

where the filter coefficients a_i and b_i are obtained by computing the z-transform of the IIR filter $\Gamma_{\theta_p}(n,m) = \theta_p(n) \, \mathbf{e}^{j\frac{2\pi n m}{M}}$. Combined with this recursive implementation of the TFR, the proposed method allows for real-time estimation of both the IF and IA.

VI. Results

Our numerical experiments[1] are made with the ASTRES toolbox [5].We consider two distinct STFTs with $M = 500$ frequency bins, respectively computed with a Gaussian analysis window $\theta(n) = \frac{1}{\sqrt{2\pi}L} \mathbf{e}^{-n^2/L^2}$, such that $g(m) = \frac{2\sqrt{\pi}L}{M} \mathbf{e}^{-(\frac{2\pi m L}{M})^2}$, and the recursive version (c.f. Section V), both computed with parameters $L = 20$ and $p = 3$.

A. IF Estimation

We consider a synthesized signal of length $N = 500$ made of three modes: a sinusoid, a linear chirp and a sinusoidally-FM chirp. We control the Signal-to-Noise Ratio (SNR) by adding white Gaussian noise to the MCS. Overlaping components scenario is out of the scope of this paper, as a similar experiment has been carried out in [26].

We assess the IF estimation performance of the proposed approaches denoted FRI TLS and Recursive FRI, applied on the corresponding computed STFTs. Additionally, we use the Vertical Synchrosqueezing Transform (VSST) to sharpen the resulting (non-recursive) TFR, denoted FRI SST when applied on the squared-moduled synchrosqueezed STFT instead of its spectrogram. For the FRI SST method, we arbitrary use for g a Gaussian function with a standard deviation equal to 0.5, which empirically provides the best correlation with the data. In Fig. 1, we compare our results with the state-of-the-art approaches: (i) Brevdo [9], (ii) Pseudo-Bayesian (PB) [17] (setting the variance of the Gaussian random walk to 2), (iii) Ridge Detector (RD) [11] (setting the frequency derivative constraints to $\lambda_s = 0.2$ and $\beta s = 0.4$) and (iv) Classical FRI presented in Section III (denoted FRI). The estimation performance are assessed in terms of relative mean squared error, $\text{RMSE}(\bar{\phi}', \hat{\phi}') = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \frac{\left|\bar{\phi}'_k(n) - \hat{\phi}'_k(n)\right|^2}{M^2}$, where $\bar{\phi}'_k(n)$ is the ground truth IF and $\hat{\phi}'_k(n)$ the estimate provided by each method.

Fig. 1 depicts the performance of various methods at different SNR levels. FRI TLS consistently obtains superior performance, outperforming Recursive FRI which uses a causal non-symmetrical analysis window, especially at high SNR. FRI SST shows less-than-optimal results except for SNR above 20dB. In contrast, the RD and Brevdo methods display behavior similar to FRI TLS at SNR levels above 2dB and −4dB, respectively, but are less effective at lower SNRs. The PB methods aim for robustness through meticulous hyperparameter selection, resulting in performance that falls short at high SNR. However, our proposed approach shows slightly improved performance at low SNR, closely tied to the Total Least-Squares (TLS) alternative. This entails selecting time-frequency points corresponding to the mean between the two highest maxima in the frequency plane. As noise spreads in the spectrogram, this mean converges to $\frac{M}{2}$ as the SNR decreases. It's noteworthy that FRI TLS shows robustness to variations in the assumed filtering kernel g in

[1] Matlab code freely available at https://github.com/QuentinLEGROS/EUSIPCO2024/

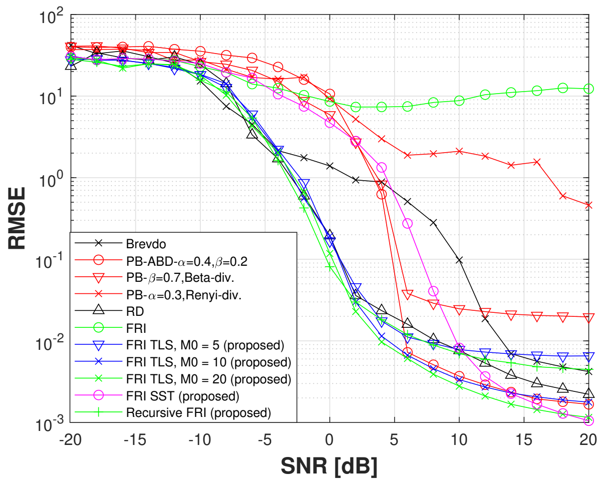

Fig. 1. RMSE of the normalized IF (averaged over 100 realizations of noise) obtained with the different competing methods for a varying SNR.

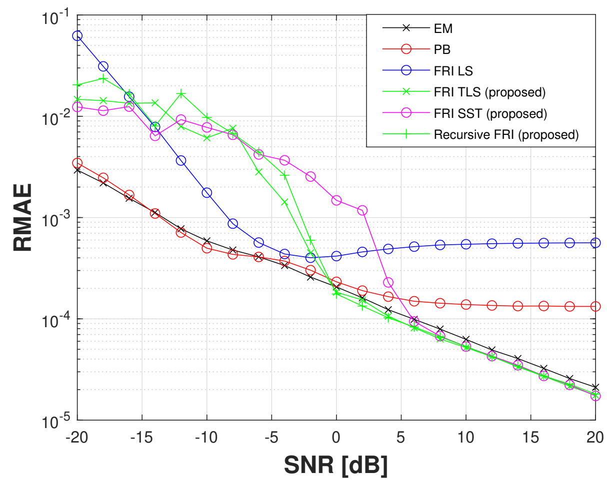

Fig. 2. RMAE of the IA (averaged over 100 realizations of noise) obtained with the different competing methods for a varying SNR

the presence of FM components broadening the observed ridges along the frequency axis [24]. Additionally, applying the proposed method on VSST yields degraded performance for SNR$<$ 8dB. Finally, we notice that the choice of M_0 has a minimal impact on estimation performance, albeit with an increased computational cost. The best results are achieved with $M_0 = 20$.

B. IA Estimation

In Fig. 2, we comparatively evaluate the IA estimation performance of the proposed FRI method considering different estimations of the IF. The FRI LS corresponds to the baseline estimator using Eq. (10). FRI TLS, FRI SST and Recursive FRI, use Eq. (12) combined with the IF estimated using the corresponding method evaluated in Section III. Our methods are compared with two state-of-the-art Bayesian techniques (i) PB (using an alpha-beta divergence with $\alpha = 0.4$, $\beta = 0.2$) [17] and (ii) Expectation Maximization (EM) [18] (using a Laplacian prior model with weight $\lambda = 10^{-2}$). To ensure a fair assessment of the methods, we now consider a mono-component signal ($N = 500$) made of a linear chirp with a linearly growing amplitude varying from 0.5 to 1. The results are expressed in terms of Relative Mean Absolute Error $\mathrm{RMAE}(\bar{\alpha}, \hat{\alpha}) = \frac{1}{NK}\sum_{k=0}^{K-1}\sum_{n=0}^{N-1}|\bar{\alpha}_k(n) - \hat{\alpha}_k(n)|$, where $\bar{\alpha}$ is the ground truth IA and $\hat{\alpha}$ the estimation. According to Fig. 2, all the proposed FRI-TLS and Recursive methods obtain satisfying results at SNR above 0dB except the baseline FRI LS method based on Eq.(11) that is biased but more robust to noise in the -15dB to 0dB range. We notice that the FRI SST only obtains satisfying results at SNR above 5dB (due to a second-order derivative-based IF estimator). As expected, the EM and PB methods are more robust at low SNRs since they were designed to deal with noise. Thus, they significantly outperform our new proposed methods only at a low SNR below 0dB. At high SNRs (above 0dB), we observe slightly better results of the proposed methods (all are mostly equivalent) compared to the robust alternatives with have a significantly higher computational costs.

C. Computation Time

Table I presents the measured computation time of the compared methods applied to the three-component signal considered in the experiments conducted to obtain Fig. 1, with $N = 500$ and various frequency resolutions indexed by M. All experiments were conducted using Matlab R2023b on a Intel(R) Core(TM) i7-12700H @ 2.30 GHz. The time corresponds to the execution of the main algorithms and does not include the computation of the TFRs. Since, all the compared methods are not designed for IA estimation, the computation time of the FRI-based methods presented in Table I only corresponds to the estimation time of IF. While the computational gain is implementation-dependent and may be further improved from parallel programming, our proposed method exhibits a significant speed-up compared to others [18], across all frequency resolutions. In terms of computation time, the proposed FRI methods outperform other methods, providing the fastest results. Additionally, the proposed method's performance is only slightly affected by the frequency resolution.

TABLE I
COMPUTATION TIME IN SECONDS (LOWER IS BETTER) OF THE COMPETING APPROACHES FOR SYNTHETIC DATA ANALYSIS, AVERAGED OVER 50 REALIZATIONS ($N = 500$).

M	500	1000	2000
Brevdo [9]	0.13	0.14	0.18
EM [18]	6.93	29.36	116.42
PB [17]	0.23	0.36	0.61
RD [11]	0.41	0.54	1.20
FRI TLS (proposed)	**0.12**	**0.12**	0.2
Recursive FRI (proposed)	**0.12**	0.13	**0.13**

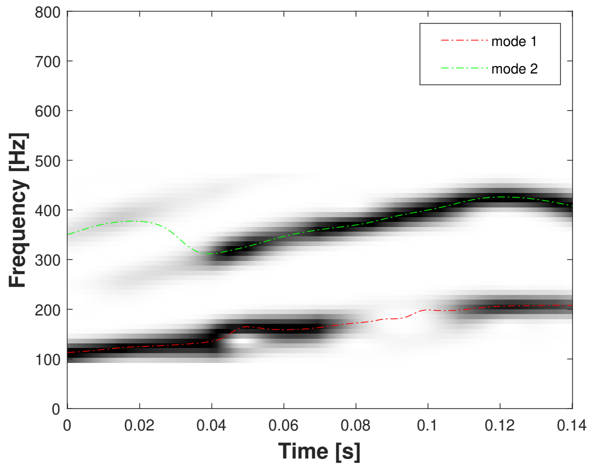

Fig. 3. Estimation of the first $K = 2$ signal components of the speech signal using the proposed TLS method.

D. Real-world signal analysis

We finally examine a real-world signal defined on a restricted segment of the time axis, as illustrated in Fig. 3. However, for the sake of readability and to evaluate the behavior of the TLS method in this scenario (with STFT computed using $L = 40$), we assume the presence of only two components. The transition between the two segments of mode 2 is smooth, as the method seeks the filtered DP that exhibits the strongest correlation with the data. Minimizing the ℓ_2-norm, as elaborated in Section III, involves selecting the TF points that minimize the mean-square error, corresponding to the mean between the two ridges of mode 1.

VII. Conclusion

This study introduces a novel observation model for estimating the instantaneous frequency and amplitude of modes within a multicomponent signal in the presence of noise. The spectrogram time slices are viewed as noisy filtered and sampled streams of Dirac pulses, thereby reducing the problem of retrieving FRI signals. Given the noise challenges, efficient use of the Prony method is hindered. Therefore, we explored alternatives like total least square estimation and a designed amplitude estimation. Our proposed approach not only achieves state-of-the-art estimation performance but also enables real-time sequential estimation through a recursive implementation of the short-time Fourier transform. Future work should focus on extending the method to handle extreme scenarios, such as overlapping ridges or high-frequency modulation, considering new practical applications.

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