Statistical Assessment of Abrupt Change Detectors for Non-Intrusive Load Monitoring

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   - General framework

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   - Definition
   - Mathematical problem statement
   - Algorithms implementation
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     - CUSUM algorithm
     - BIC algorithm

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**NILM goals**

**NILM**: Process to estimate the energy consumed by individual Home Electrical Appliances (HEAs) with a single meter in a house electrical panel connected at the PCC.

- Partition of the load curve into its main components
- Assignment of energy expenses per HEA
General framework of supervised NILM methods

Data Acquisition at the PCC

\[ v(t), i(t) \]

Data Processing & Features Extraction

Event based Approach

Event Detection

Pattern Matching of HEA

Individual HEA consumption estimation in the load curve

Non-Event based Approach

Optimization

Database of known HEAS
**General framework of supervised NILM methods**

- **Data Acquisition at the PCC**
  - $v(t), i(t)$

- **Data Processing & Features Extraction**

- **Event based Approach**
  - Event Detection

- **Non-Event based Approach**
  - Optimization

- **Database of known HEAS**

- **Pattern Matching of HEA**

- **Individual HEA consumption estimation in the load curve**
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Fast transition that occurs between stationary states in a signal

$\Rightarrow$ **NILM:** On/Off and multiple operation modes appliances
Abrupt change definition

- Fast transition that occurs between stationary states in a signal
  ⇒ **NILM**: On/Off and multiple operation modes appliances
Abrupt change definition

- Fast transition that occurs between stationary states in a signal
  ⇒ **NILM**: On/Off and multiple operation modes appliances
Abrupt change definition

- Fast transition that occurs between stationary states in a signal
  \[\Rightarrow \textbf{NILM}: \text{On/Off and multiple operation modes appliances}\]

- Need of **tools** to decide whether a change occurs or not in the signal
Mathematical formulation 1/2

- \( X_n = \{ x_m \in \mathbb{R}, m = n - k + 1, \ldots, n \} \): vector of the last \( k \) available samples of a signal at the current time \( n \).
- \( x_m \) follows a probability density function (PDF) \( p_\theta(x_m) \) depending on a deterministic parameter \( \theta \).
- **Abrupt change**: modification of \( \theta \) at a change time \( n_c \).

\[ X_n = \{ x_m \in \mathbb{R}, m = n - k + 1, \ldots, n \} \]

\( \Rightarrow \) Hypothesis Test:
- \( H_0: \text{"no change"} \) versus \( H_1: \text{"with a change at time } n_c \)"
Mathematical formulation 2/2

under $H_0$, $\theta = \theta_0$ for $n - k + 1 \leq m \leq n$

under $H_1$, $\theta = \begin{cases} 
\theta_{1a} & \text{for } n - k + 1 \leq m \leq n_c - 1 \\
\theta_{1b} & \text{for } n_c \leq m \leq n 
\end{cases}$

$\Rightarrow$ Decision rule:
At each time $n$, comparison of a decision function $g_n$ to a threshold value $h$ adjusted according to decision probabilities

- decide $H_1$ if $g_n > h$
- decide $H_0$ if $g_n \leq h$
⇒ Assessment of all the algorithms in strictly the same conditions

⇒ Sliding window of $k = 5$ samples for the three detection algorithms to be studied:
  ◦ The Effective Residual algorithm
  ◦ The CUMulative SUM (CUSUM) algorithm
  ◦ The Bayesian Information Criterion (BIC) algorithm
Effective Residual

- Parity equation-based approach:
  ⇒ temporal redundancies of measurements
  ⇒ used for sensor fault detection and isolation
Effective Residual decision function

- Absolute variation $\delta_m$ between 2 consecutive signal samples:
  \[ \delta_m = |x_m - x_{m-1}| \quad \text{for} \quad n - k + 2 \leq m \leq n \]

- Residual $r_m$: difference between 2 consecutive variations
  \[ r_m = |\delta_m - \delta_{m-1}| \quad \text{for} \quad n - k + 3 \leq m \leq n \]

- Effective Residual decision function: sum of the last 3 residuals
  \[ g_n = r_n + r_{n-1} + r_{n-2} \]

\[ \begin{align*}
H_1 & \quad \geq h \\
H_0 &
\end{align*} \]

⇒ Detection of a mean change at $n_c = n$ from the last $k = 5$ samples of the signal
CUSUM algorithm

- Used for biomedical engineering as well as for NILM applications
- Based on **log-likelihood ratio maximization** over change time $n_c$
- Most common form: statistical test for the detection of a **mean change** in a Gaussian process $\mathcal{N}(\mu, \sigma)$
CUSUM algorithm principle

- PDFs of $X_n$ under hypotheses $H_0$ and $H_1$:

$$p(X_n|H_0) = \prod_{m=n-k+1}^{n} p_{\theta_0}(x_m)$$

$$p(X_n|H_1) = \prod_{m=n-k+1}^{n-1} p_{\theta_{1a}}(x_m) \prod_{m=n_c}^{n} p_{\theta_{1b}}(x_m)$$

- If $\theta_0=\theta_{1a}$, log-likelihood ratio $L(X_n, n_c)$:

$$L(X_n, n_c) = \ln \left( \frac{p(X_n|H_1)}{p(X_n|H_0)} \right) = \sum_{m=n_c}^{n} s_m \quad \text{with} \quad s_m = \ln \left( \frac{p_{\theta_{1b}}(x_m)}{p_{\theta_{1a}}(x_m)} \right)$$

- CUSUM decision rule: maximization of the log-likelihood ratio over $n_c$

$$g_n \begin{cases} H_1 \\ H_0 \end{cases} h, \quad \text{with} \quad g_n = \max_{n-k+1 \leq n_c \leq n} \sum_{m=n_c}^{n} s_m$$
CUSUM decision function

■ Changing parameter: mean value $\mu$ in a Gaussian process $\mathcal{N}(\mu, \sigma)$

under $H_0$, $\mu = \mu_0$ for $n-k+1 \leq m \leq n$

under $H_1$, $\mu = \begin{cases} \mu_1a & \text{for } n-k+1 \leq m \leq n_c-1 \\ \mu_1b & \text{for } n_c \leq m \leq n \end{cases}$

■ Instantaneous log-likelihood ratio $s_m$:

$$s_m = \frac{(x_m - \mu_{1b})^2}{2\sigma^2} + \frac{(x_m - \mu_{1a})^2}{2\sigma^2} = \frac{\Delta\mu}{\sigma^2} \left( x_m - \frac{\mu_{1b} + \mu_{1a}}{2} \right)$$

with $\Delta\mu = \mu_{1b} - \mu_{1a}$

⇒ The CUSUM decision rule $g_n$:

For an abrupt change occurring at $n_c = n$ in a sliding window of $k = 5$ samples

with $\hat{\mu}_{1b} = x_n$, $\hat{\mu}_{1a} = \frac{1}{4} \sum_{m=n-4}^{n-1} x_m$ and $\hat{\sigma}^2 = \frac{1}{4} \sum_{m=n-4}^{n-1} (x_m - \hat{\mu}_{1a})^2$,

$$g_n \begin{cases} H_1 & \text{if } g_n \geq h \\ H_0 & \text{if } g_n < h \end{cases}$$

with $g_n = s_n = \frac{(x_n - \hat{\mu}_{1a})^2}{2\hat{\sigma}^2}$
BIC algorithm

- Used for acoustic change detection
- Division of the sequence of observed random samples into homogeneous segments by performing a hypothesis test at each potential change point
  - Hypothesis $H_0$: on both sides of this point, the signal follows the same probabilistic model
  - Hypothesis $H_1$: a model change occurs
BIC algorithm principle

■ The BIC of $X_n$ under hypothesis $H_i$, $i \in \{0, 1\}$: likelihood criterion penalized by the model complexity

$$
\text{BIC}(H_i) = \ln(p(X_n|H_i)) - \frac{\lambda}{2} M \ln(k)
$$

$p(X_n|H_i)$ Maximized data likelihood for the given model

$\lambda$ Penalty factor (ideally equal to 1)

$k$ Last available samples

$M$ Number of parameters in the probabilistic model

■ Probabilistic model:

$$
\begin{align*}
H_0 & : \quad x_{n-k+1}, \ldots, x_n \sim \mathcal{N}(\mu_0, \sigma_0) \\
H_1 & : \quad x_{n-k+1}, \ldots, x_{n_c-1} \sim \mathcal{N}(\mu_{1a}, \sigma_{1a}); \\
& \quad x_{n_c}, \ldots, x_n \sim \mathcal{N}(\mu_{1b}, \sigma_{1b})
\end{align*}
$$

■ Model parameters:

- Under $H_0$: $\mu_0$ and $\sigma_0$ ($M = 2$)
- Under $H_1$: $\mu_{1a}, \sigma_{1a}$, and $\mu_{1b}, \sigma_{1b}$ ($M = 4$)

$\Rightarrow$ Maximization of $\text{BIC}(H_i)$ when $\mu$ and $\sigma^2$ are replaced by their MLEs $\hat{\mu}$ and $\hat{\sigma}^2$
BIC decision function

- The BIC decision function $g_n$ is:

$$g_n \begin{cases} H_1 & \geq h \\ H_0 \end{cases} \text{ with } g_n = \max_{n-k+1 \leq n_c \leq n} \Delta BIC(n_c)$$

where $\Delta BIC(n_c) = BIC(H_1) - BIC(H_0)$

$$= \frac{k}{2} \ln(\hat{\sigma}_0^2) - \frac{(n_c - n + k - 1)}{2} \ln(\hat{\sigma}_{1a}^2) - \frac{(n - n_c + 1)}{2} \ln(\hat{\sigma}_{1b}^2) - \lambda \ln(k)$$

$\Rightarrow$ BIC decision rule:

For an abrupt change occurring at $n_c = n - 1$ in a sliding window of $k = 5$ samples:

$$g'_n \begin{cases} H_1 & \geq h' \\ H_0 \end{cases} \text{ with } g'_n = \frac{1}{2} \ln\left(\frac{\hat{\sigma}_{10}^2}{\hat{\sigma}_{1a}^6 \hat{\sigma}_{1b}^4}\right), \ h' = h + \lambda \ln(5)$$

with $\hat{\sigma}_0^2 = \frac{1}{5} \sum_{m=n-4}^{n} (x_m - \hat{\mu}_0)^2$, $\hat{\sigma}_{1a}^2 = \frac{1}{3} \sum_{m=n-4}^{n-2} (x_m - \hat{\mu}_{1a})^2$ and $\hat{\sigma}_{1b}^2 = \frac{1}{2} \sum_{m=n-1}^{n} (x_m - \hat{\mu}_{1b})^2$
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Test bench

- Monte Carlo Test repeated 100,000 times
  
  \[ X_n \text{ is filled with } 5 \, i.i.d \, \text{samples } x_m \sim \mathcal{N}(0, \sigma) \text{ with } \sigma = 1 \]
  
  \[ \text{Under } H_1, \text{ addition of } \Delta \mu = \text{SNR} \times \sigma:\]
  
  - to the last sample for Effective Residual and CUSUM
  - to the last 2 samples for the BIC

⇒ Assessment made for:
  
  - fixed SNR values
  - varying SNR values ranging from 0.5 to 10

⇒ Use of 400 logarithmically spaced values of \( h \)
Performance metrics

■ Basic performance metrics:

- True Positive TP → detection of a change when there is really one
- True Negative TN → no detection of a change when there is not
- False Positive FP → detection of a change when there is not
- False Negative FN → no detection of a change when there is really one

■ Computation of performance rates:

True Positive Rate TPR

\[ TPR = \frac{TP}{TP + FN} \]

False Positive Rate FPR

\[ FPR = \frac{FP}{TN + FP} \]

Precision \( P_R \)

\[ P_R = \frac{TP}{TP + FP} \]
Performance metrics

- **Receiver Operating Characteristics (ROC):**
  Plot of the TPR versus the FPR for varying values of $h$

$$\lim_{h \to -\infty} TPR = 1, \quad \lim_{h \to -\infty} FPR = 1, \quad \lim_{h \to +\infty} TPR = 0, \quad \lim_{h \to +\infty} FPR = 0$$

- **SNR = 3**
- Uniformly distributed SNR between 0.5 and 10

$\Rightarrow$ CUSUM ROC curve is the closest to the “optimal” point (FPR=0; TPR=1)
Performance metrics

- **Area Under the Curve (AUC):**
  Numerical integration of the ROC curve

AUC values computed for the three considered detection algorithms

<table>
<thead>
<tr>
<th></th>
<th>Effective Residual</th>
<th>CUSUM</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR = 0.5</td>
<td>0.51</td>
<td>0.59</td>
<td>0.53</td>
</tr>
<tr>
<td>SNR = 3</td>
<td>0.75</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>SNR = 6</td>
<td>0.96</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>variable SNR</td>
<td>0.85</td>
<td>0.91</td>
<td>0.89</td>
</tr>
</tbody>
</table>

⇒ CUSUM: best detection ability

SNR-AUC profile of the Effective Residual, CUSUM and BIC algorithms.
Performance metrics

- **F-Measure** ($F_M$) harmonic mean of $P_R$ and TPR

\[
F_M = \left( \frac{P_R^{-1} + TPR^{-1}}{2} \right)^{-1} = 2 \times \frac{P_R \times TPR}{P_R + TPR}
\]

- F-measure ($F_M$) depending on threshold values $h$ of each detector:

  - For small values of $h$, $P_R = 1/2$ and $TPR = 1$, so $F_M = 2/3$.
  - For large values of $h$, $TPR = 0$, so $F_M = 0$.
  - For the three detectors, the maximum $F_M$ score is reached for a specific threshold value which is the optimal one.

**SNR = 3**

Uniformly distributed SNR between 0.5 and 10
Practical case study

- Realization of a controlled consumption scenario using our own measurement system
- Measured current $i[k]$ and voltage $v[k]$ sampled at $F_s = 1.2 \text{ kHz}$
Application of the detectors

- Active power signal \( P[n] = \frac{1}{M} \sum_{k=n-M+1}^{n} v[k]i[k] \), with \( M = \frac{F_s}{F} \)

- For each detector, \( h \) is set to allow the detection of the lowest \( \Delta P = (P[n] - P[n-1]) \) (low-energy lamp switch-on)

- Power time profile and detection results of each algorithm:
Detection results 1/2

Threshold values $h$ and number of TP, FP and $P_R$ of the three considered detection algorithms applied to the active power signal.

<table>
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<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>31.0</td>
<td>850.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Number of detected events</td>
<td>32</td>
<td>8</td>
<td>33</td>
</tr>
<tr>
<td>TP</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>FP</td>
<td>25</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>$P_R$</td>
<td>21.3%</td>
<td>87.5%</td>
<td>21.2%</td>
</tr>
</tbody>
</table>

The CUSUM algorithm appears to be the most effective with a high $P_R$ and a small number of false positive.
Detection results 2/2

- Relatively large number of detected events for the Effective Residual and the BIC algorithms
  - Noisy steady-states leading to false positive $F_P$ and inaccurate detections.
- Zoom overview of the active power signal drawn by the iron switch on:
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Conclusion

- Event-based NILM approach

- Study of 3 detectors: **Effective Residual**, **CUSUM** and **BIC**

- Test bench for the **statistical assessment** of the detectors under the same conditions

- Definition of **metrics** to judge the detectors performances

- **Practical case study**: application of the detectors to a controlled HEAs consumption scenario.
Prospectives

- Threshold setup from the Probability Density Function of the decision functions
- Extension to multidimensional signals to make a decision from several features
Thank you for your attention.